UNIT – III

Digital Modulation Techniques: Wave form representation of different digital modulation techniques; Amplitude Shift Keying, Coherent Phase Shift Keying(PSK)- Binary Phase Shift Keying, Quadrature Phase Shift Keying, Differential PSK, Coherent Frequency Shift Keying, Probability of error for BASK, BPSK, BFSK.

Wave form representation or A line code is the code used for data transmission of a digital signal over a transmission line. This process of coding is chosen so as to avoid overlap and distortion of signal such as inter-symbol interference.

Properties of Line Coding

Following are the properties of line coding −

As the coding is done to make more bits transmit on a single signal, the bandwidth used is much reduced.

For a given bandwidth, the power is efficiently used.

The probability of error is much reduced.

Error detection is done and the bipolar too has a correction capability.

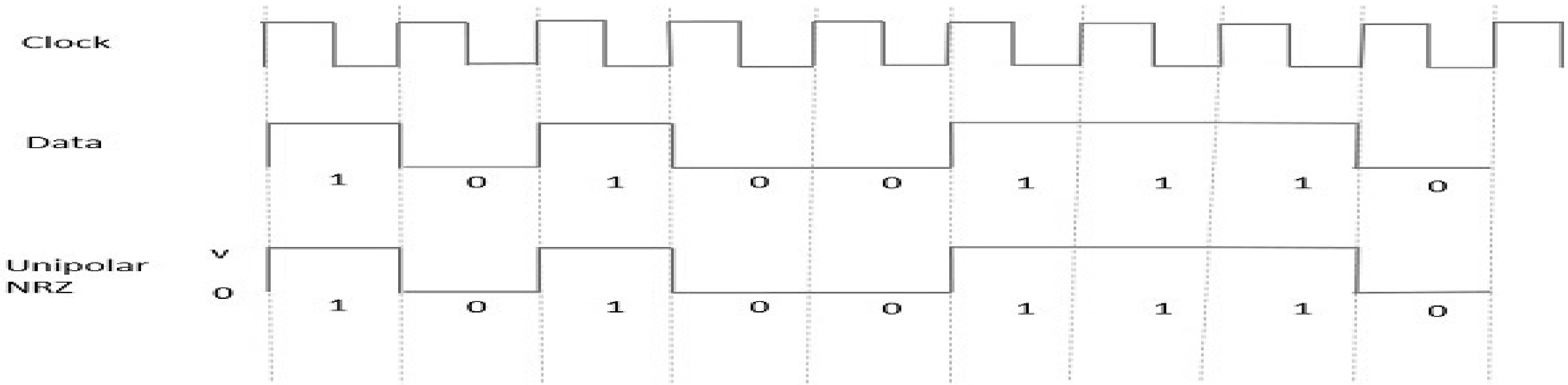
Power density is much favorable.

The timing content is adequate.

Long strings of **1s** and **0s** is avoided to maintain transparency.

Types of Line Coding: There are 3 types of Line Coding i) Unipolar Polar ii) Polar iii)Bi-polar

Unipolar Non-Return to Zero *NRZ*



Advantages

The advantages of Unipolar NRZ are −i) It is simple. ii) A lesser bandwidth is required.

Disadvantages

The disadvantages of Unipolar NRZ are −i) No error correction done.

ii)Presence of low frequency components may cause the signal droop.

iii)No clock is present.

iv)Loss of synchronization is likely to occur (especially for long strings of **1s** and **0s**).

Line codes for the electrical representations of binary data.

(a) Unipolar non-return-to-zero (NRZ) signaling (on-off signalling).

- Binary 1 – pulse, Binary 0 – no pulse

(b) Polar NRZ signaling.

- Binary 1 – Positive pulse, Binary 0 – negative pulse

(c) Unipolar return-to-zero (RZ) signaling.

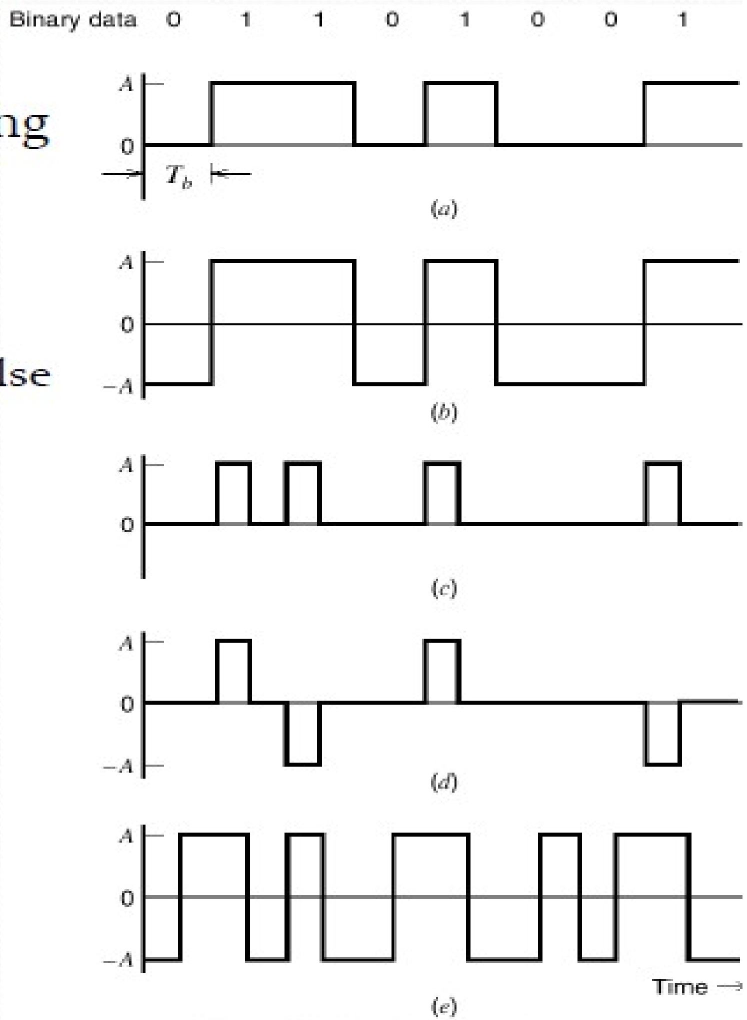
- Binary 1 – Positive half width pulse , Binary 0 – no pulse

(d) Bipolar RZ signaling.

- Binary 1 – Alternative + and - pulses, Binary 0 – no pulse

(e) Split-phase or Manchester code.

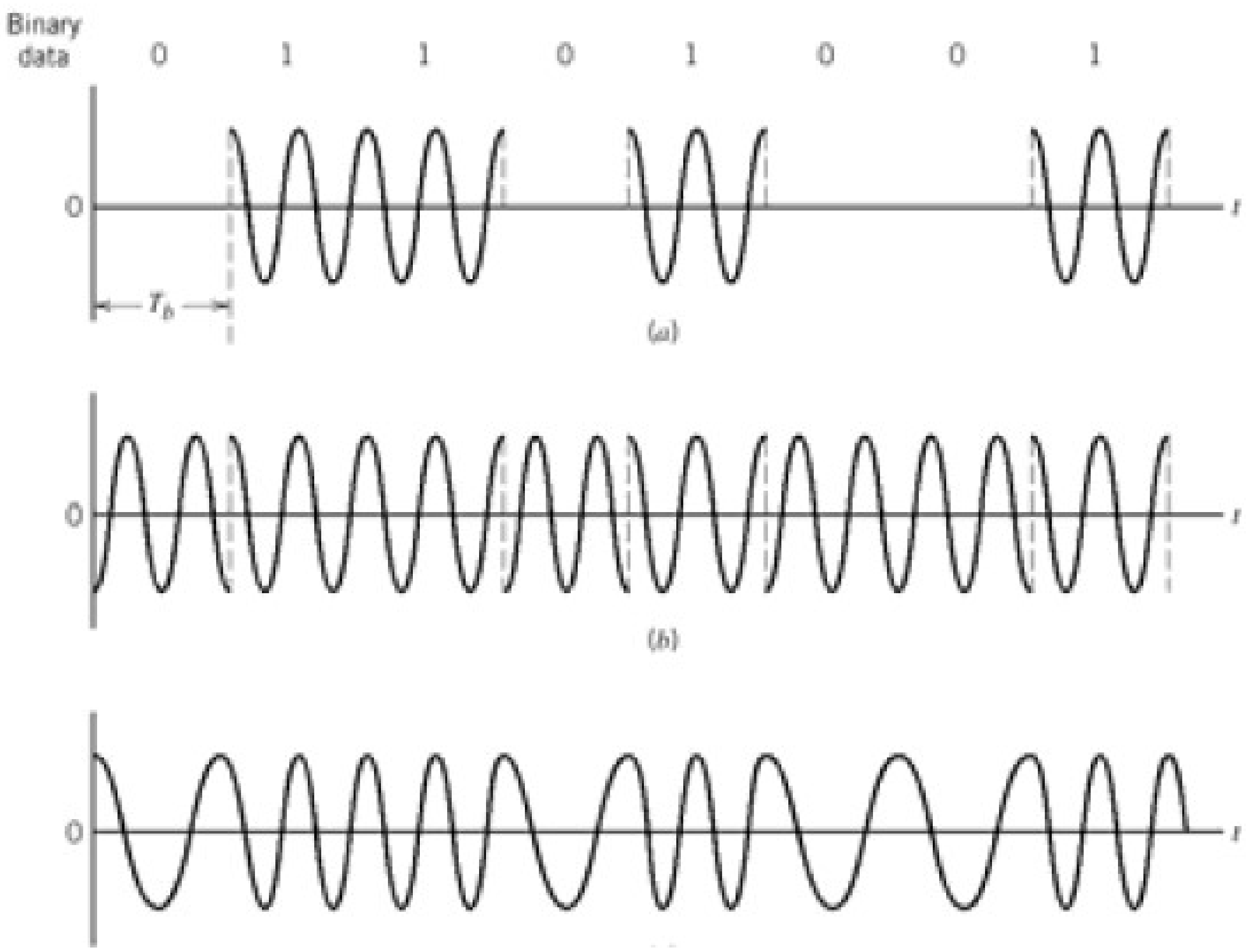
- Binary 1 – Positive to negative transition pulse with half symbol width, Binary 0 – Negative to positive pulse with half symbol width



Digital Modulation

- In binary signaling, the modulator produces one of two distinct signals in response to bit of source data at a time. By shifting the parameters either Amplitude or Phase or frequency of carrier with reference to the binary data digital modulation can be achieved.
- Binary modulation types
  - Binary ASK
  - Binary PSK (BPSK)
  - Binary FSK
  - Binary QPSK
  - DPSK
- Coherent Detection
- Non coherent Detection

- Types
- ASK
- PSK
- FSK

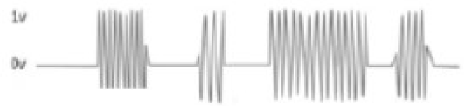


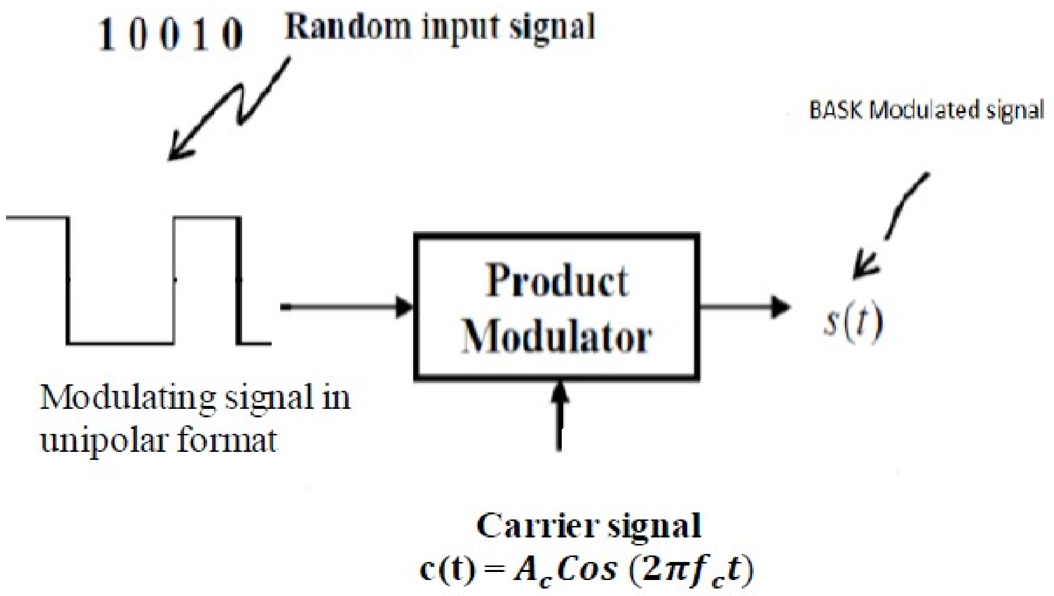
Binary Amplitude Shift Keying

- In transmitter the binary data sequence is given to an on-off encoder.
- Which gives an output E_b volts for symbol 1
- and 0 volt for symbol 0.
- The resulting binary wave and sinusoidal carrier ϕ_1 (t) are applied to a product modulator. The desired BASK wave is obtained at the modulator output.
- In demodulator, the received noisy BASK signal x(t) is apply to correlator with coherent reference signal ϕ_1 (t) . The correlator output x is compared with threshold λ.
- If $x > \lambda$ the receiver decides in favour of symbol 1. If $x < \lambda$ the receiver decides in favour of symbol 0.

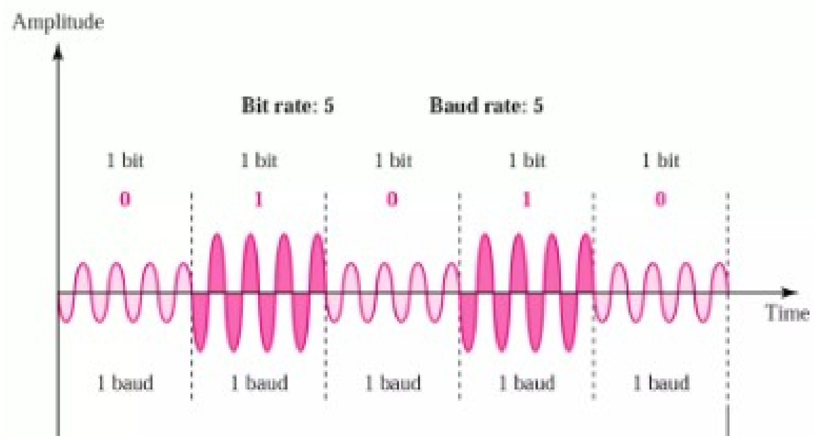
- The binary ASK may be expressed as

- $$s(t) = \begin{cases} s_1(t) = A\cos(2\pi f_c t) & For\ transmitted\ bit\ '1' \\ s_1(t) = \ 0 \text{ or } A_2 Cos\ (2\pi f_c t) & For\ transmitted\ bit\ '0' \end{cases}$$

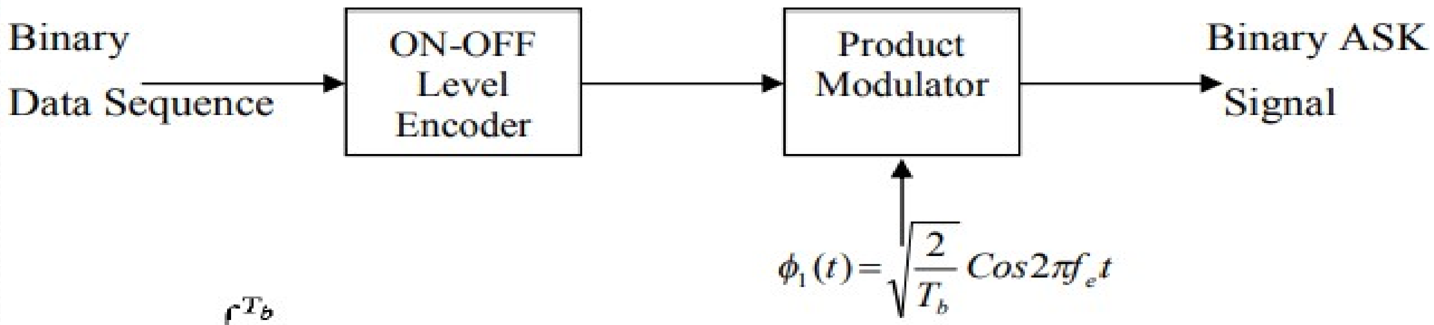




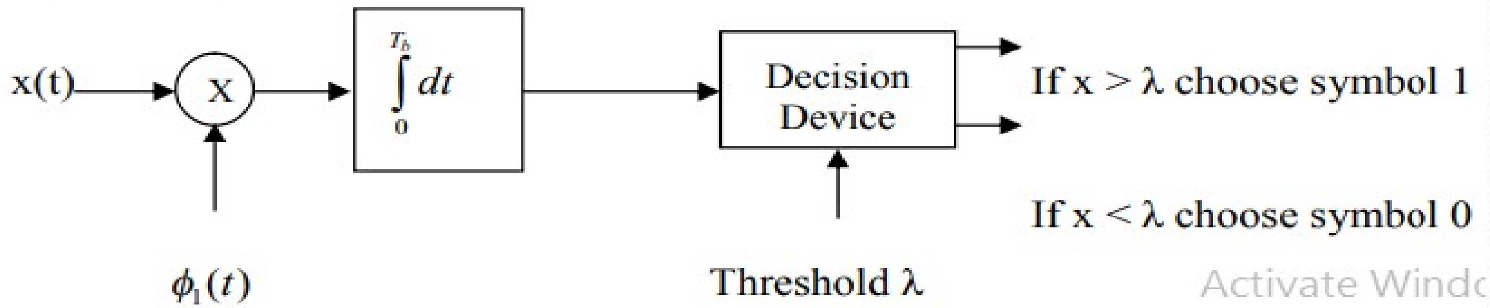
ASK(AMPLITUDE SHIFT KEYING)



- For binary '1' $$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$
- For binary '0' $s_2(t) = 0$ $\quad 0 \leq t < T_b$
- Transmitter

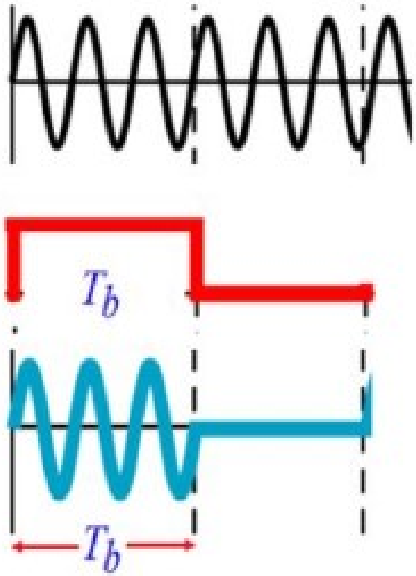


- Receiver $$x_1 = \int_0^{T_b} x(t)\phi_1(t)\, dt$$



In digital communication literature, the usual practice is to assume that the carrier $c(t) = A_c \cos 2\pi f_c t$ has unit energy measured over one symbol (bit) duration.

The power of the carrier signal $P_c = \dfrac{A_c^2}{2}$ watts.



Energy for a duration T_b pulse $E_b = T_b P_c = \dfrac{A_c^2}{2} T_b$

or $A_c = \sqrt{\dfrac{2E_b}{T_b}}$

$$c(t) = A_c \cos 2\pi f_c t = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$$

$\sqrt{\dfrac{2}{T_b}} \cos 2\pi f_c t$ referred to as basis function (Orthonormal)

Now

$$s(t)\big|_{ASK} = \sqrt{E_b}\sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) = \sqrt{E_b}\,\phi_1(t), \quad 0 \le t \le T_b$$

where $\phi_1(t)$ = orthogonal signal as is used to Rept the signal.

Then for $b(t)$ = Binary Symbol 1 = $s_1(t) = s(t)\big|_{ASK} = \sqrt{E_b}\,\phi_1(t)$

" " 0 = $s_2(t) = s(t)\big|_{ASK} = 0$. $\Big\}\ 0 \le t \le$

BASK

In BASK a sinusoidal carrier is simply gated on and off by the bit sequence to be transmitted.

A *binary amplitude-shift keying* (BASK) signal can be defined as

$$s(t) = A_c\, m(t) \cos 2\pi f_c t, \quad 0 \le t \le T_b$$

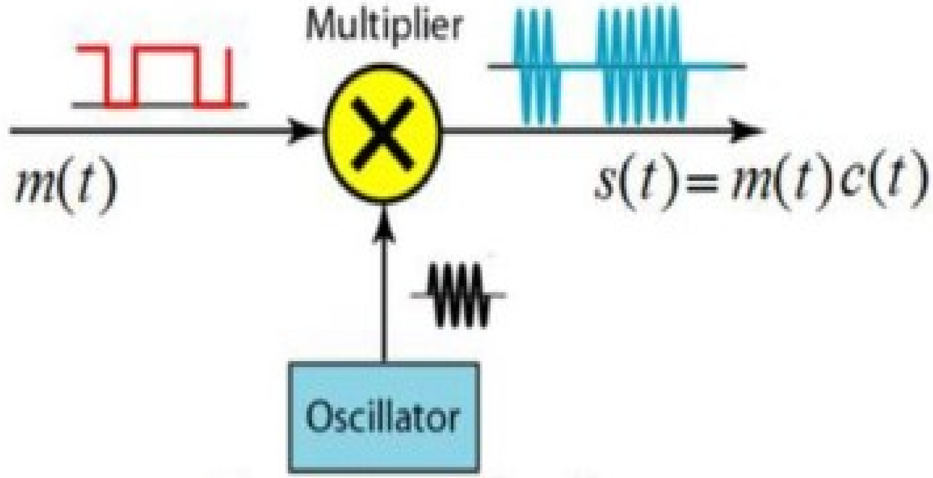
where A_c is a constant amplitude

$m(t)$ is digital information signal

$m(t)$ = 1 or 0

f_c is carrier frequency

T_b is the bit duration



$$c(t) = A_c \cos 2\pi f_c t$$

$$s(t) = \begin{cases} s_1(t) = A_c \cos 2\pi f_c t \\ \quad = \sqrt{\dfrac{2E_b}{T_b}} \cos 2\pi f_c t, \quad 0 \le t \le T_b : \text{Logic 1} \\ s_2(t) = 0, \quad\quad Else : \text{Logic 0} \end{cases}$$

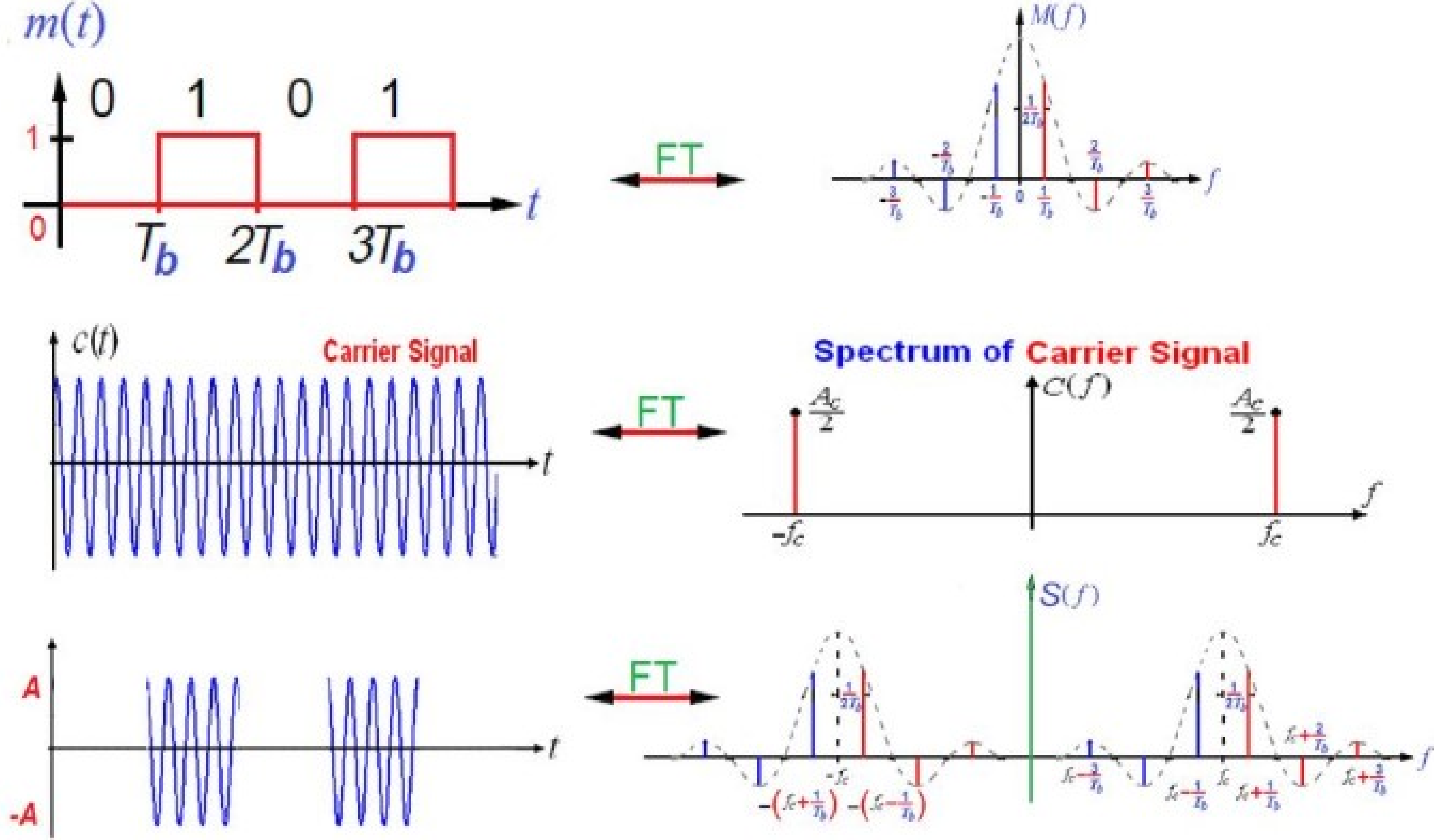
Average Energy per bit

E_b

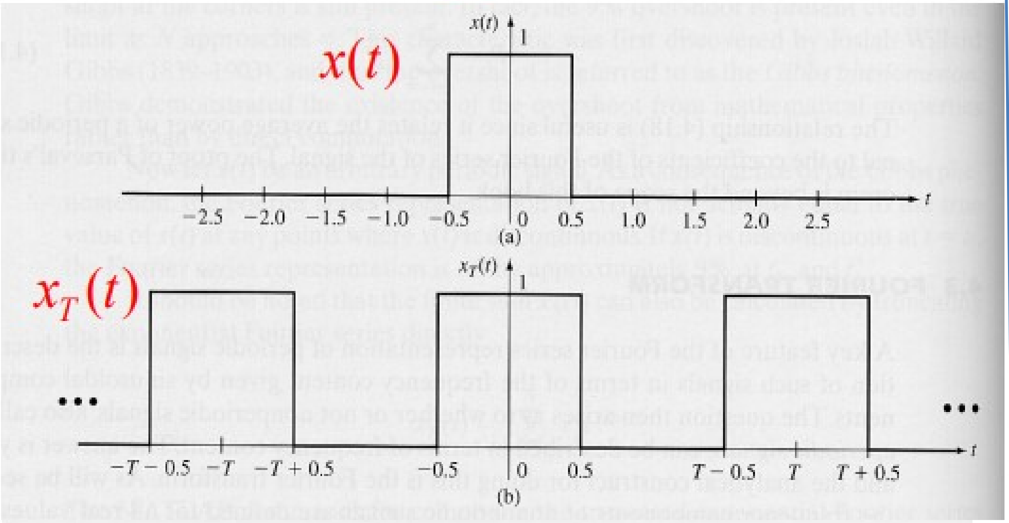
Spectrum of BASK

$$s(t) = A_c\, m(t) \cos 2\pi f_c t, \qquad 0 \le t \le T_b$$

$$S(f) = \frac{A_c}{2} M(f + f_c) + \frac{A_c}{2} M(f - f_c)$$



Frequency Content of the Rectangular Pulse



$$x(t) = \lim_{T \to \infty} x_T(t)$$

- Since $x_T(t)$ is periodic with period T, we can write

$$x_T(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}, \quad t \in \mathbb{R}$$

where

$$c_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jk\omega_o t} dt, \quad k = 0, \pm 1, \pm 2, ..$$

- For $k = 0$:

$$c_0 = 1/T$$

$\rightarrow \mathrm{sinc}(x) = \dfrac{\sin x}{x}$

- For $k = \pm 1, \pm 2, \ldots$:

$$c_k = \frac{2}{k\omega_0 T} \sin\left(\frac{k\omega_0}{2}\right) = \frac{1}{k\pi} \sin\left(\frac{k\omega_0}{2}\right)$$

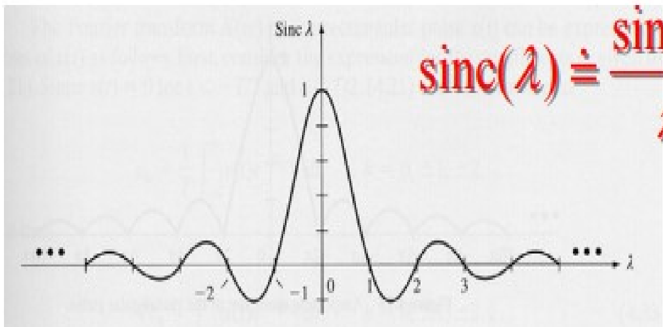
$$\omega_0 = 2\pi / T$$

- It can be easily shown that

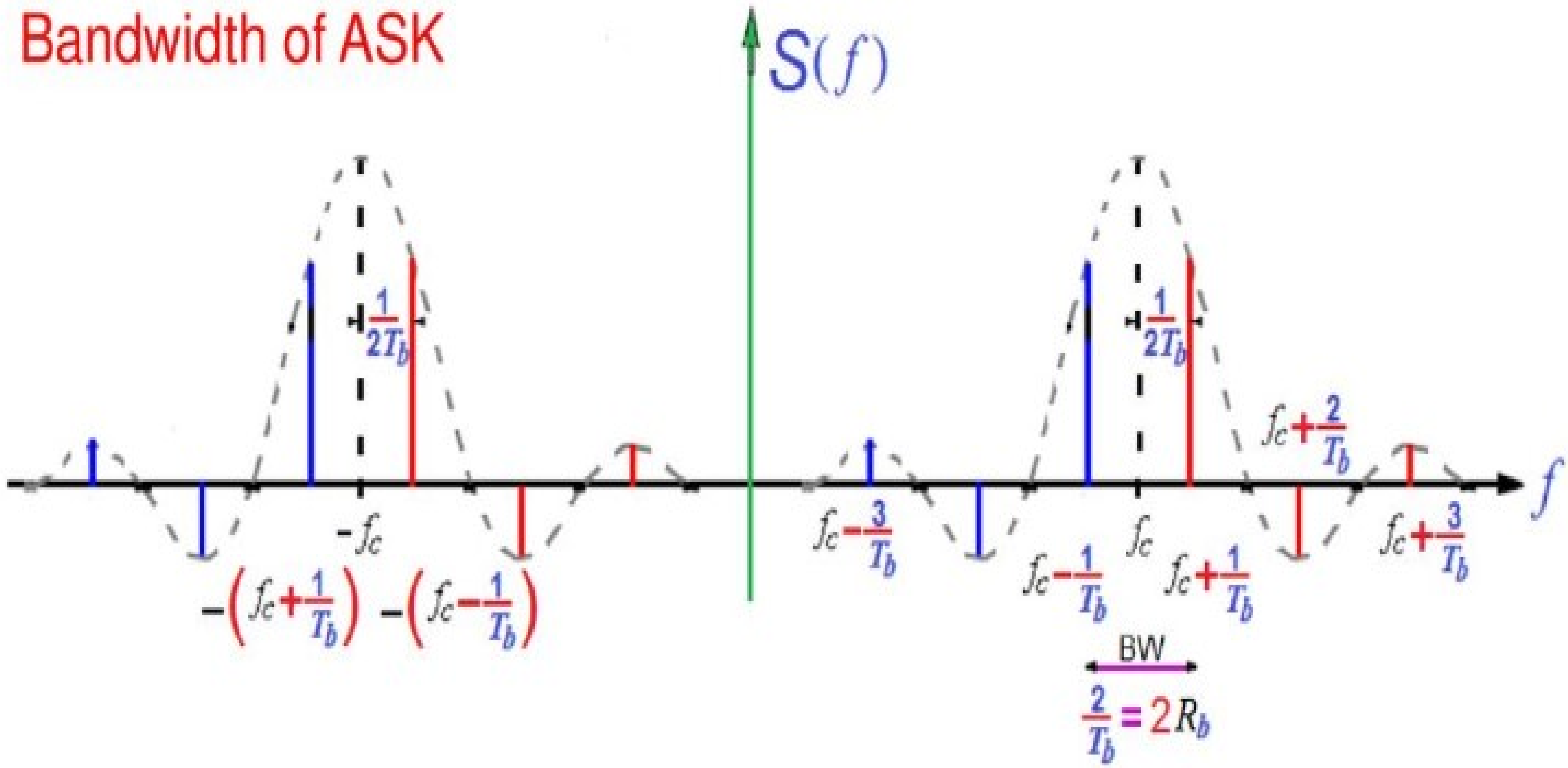
$$\lim_{T \to \infty} T c_k = \mathrm{sinc}\left(\frac{\omega}{2\pi}\right), \quad \omega \in \mathbb{R}$$

where

$$\mathrm{sinc}(\lambda) \doteq \frac{\sin(\pi\lambda)}{\pi\lambda}$$



Bandwidth of ASK



- The first, bandwidth in hertz, refers to the range of frequencies in a composite signal or the range of frequencies that a channel can pass.
- The second, bandwidth in bits per second, refers to the speed of bit transmission in a channel or link. Often referred to as Capacity

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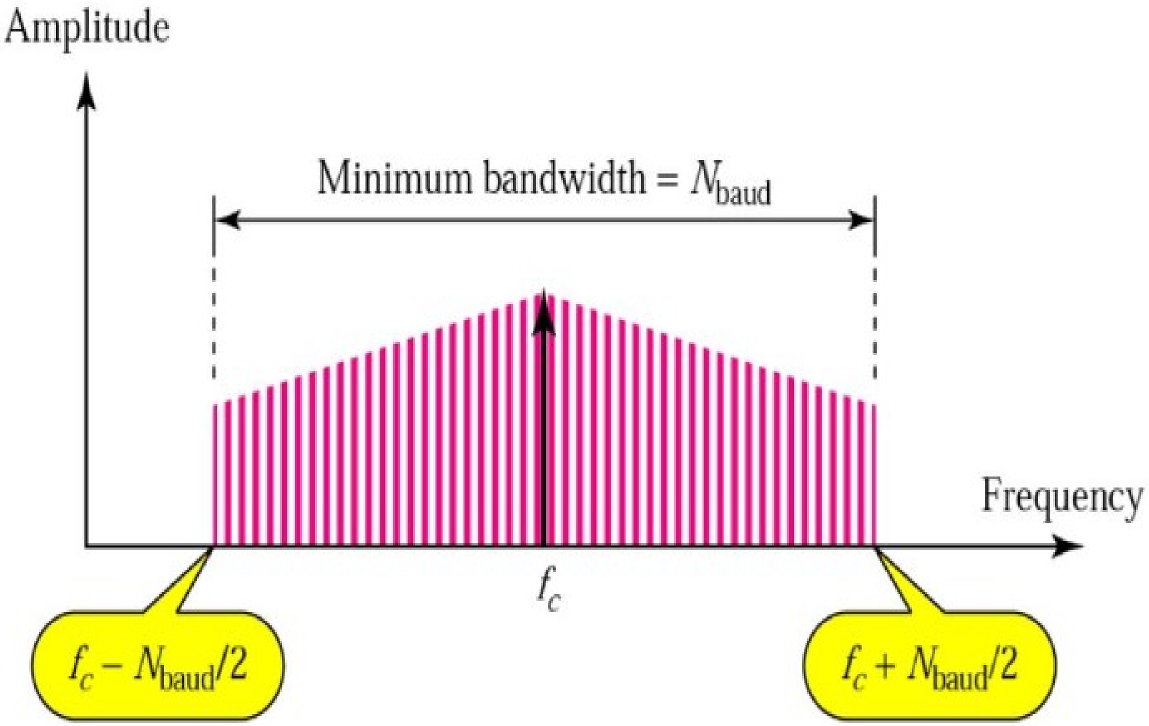
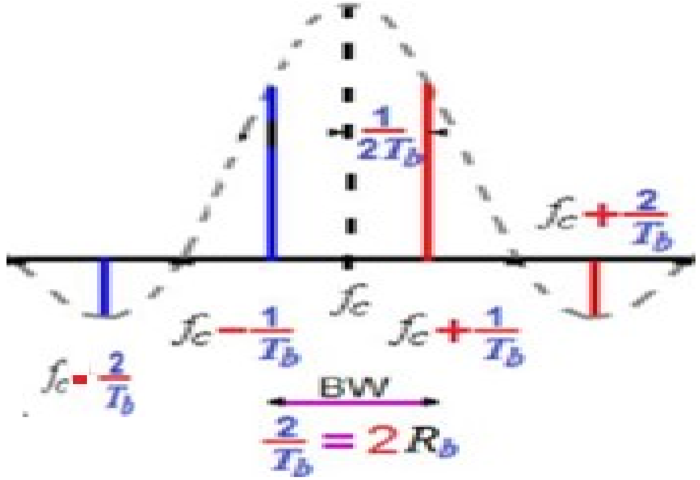




With ASK, the bit rate is equal to the minimum Nyquist bandwidth

$$BW = \frac{f_b}{1} = f_b; \quad Baud = \frac{f_b}{1} = f_b$$

Bit rate is the number of bits per second. Baud rate is the number of signal units per second. Baud rate is less than or equal to the bit rate.

Ex1: Find the minimum bandwidth for an ASK signal transmitting at 2000 bps.

Ans: In ASK the baud rate and bit rate are the same.

The baud rate is therefore 2000.

An ASK signal requires a minimum bandwidth equal to its baud rate.

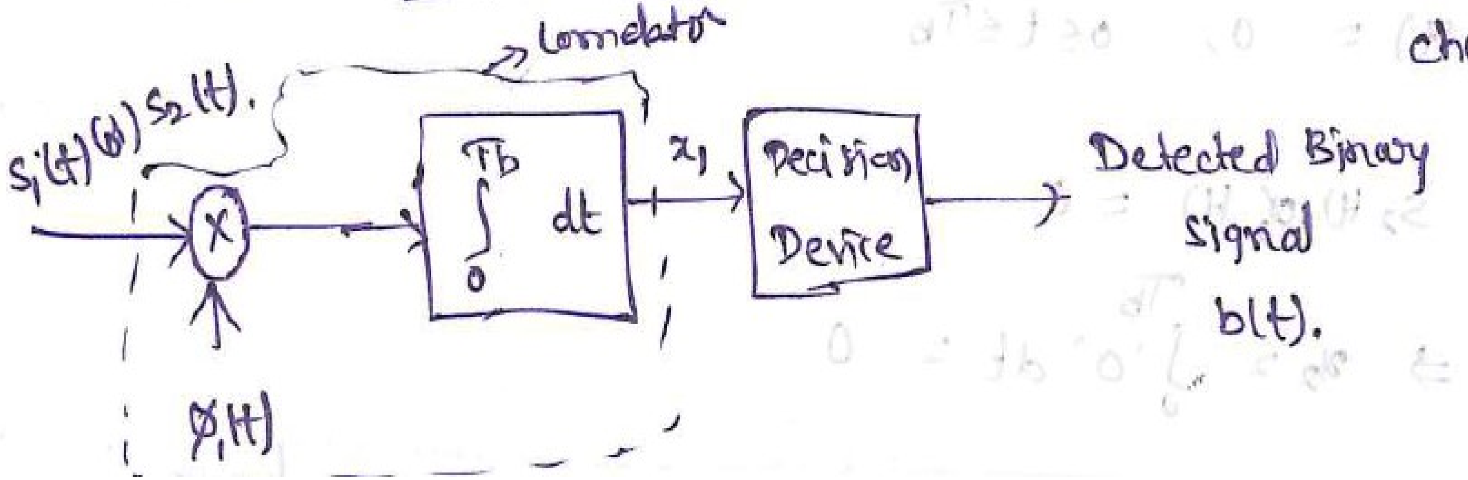
Therefore, the minimum bandwidth is 2000 Hz

Ex2: Given a bandwidth of 5000 Hz for an ASK signal, what are the baud rate and bit rate?

In ASK the baud rate is the same as the bandwidth, which means the baud rate is 5000.

But because the baud rate and the bit rate are also the same for ASK, the bit rate is 5000 bps.

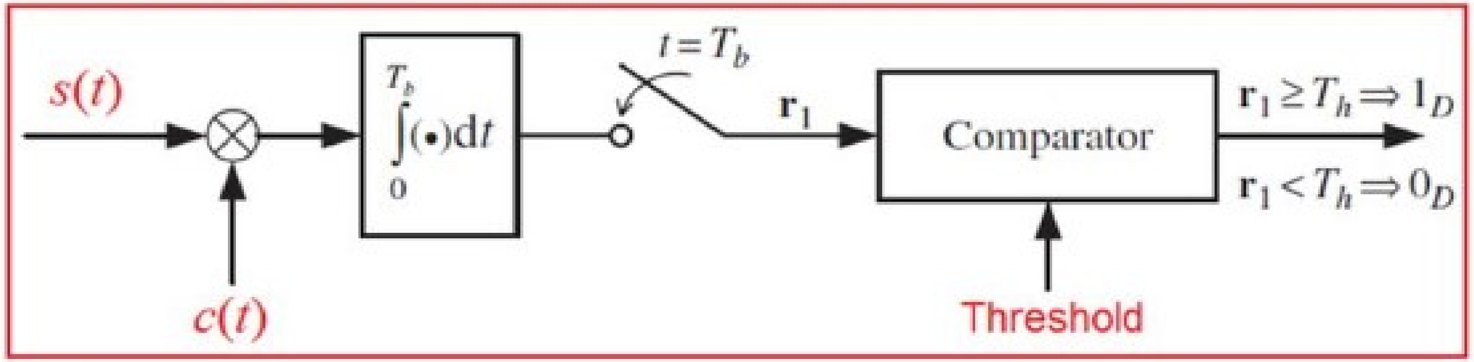
(i) Coherent ASK Demodulator :-

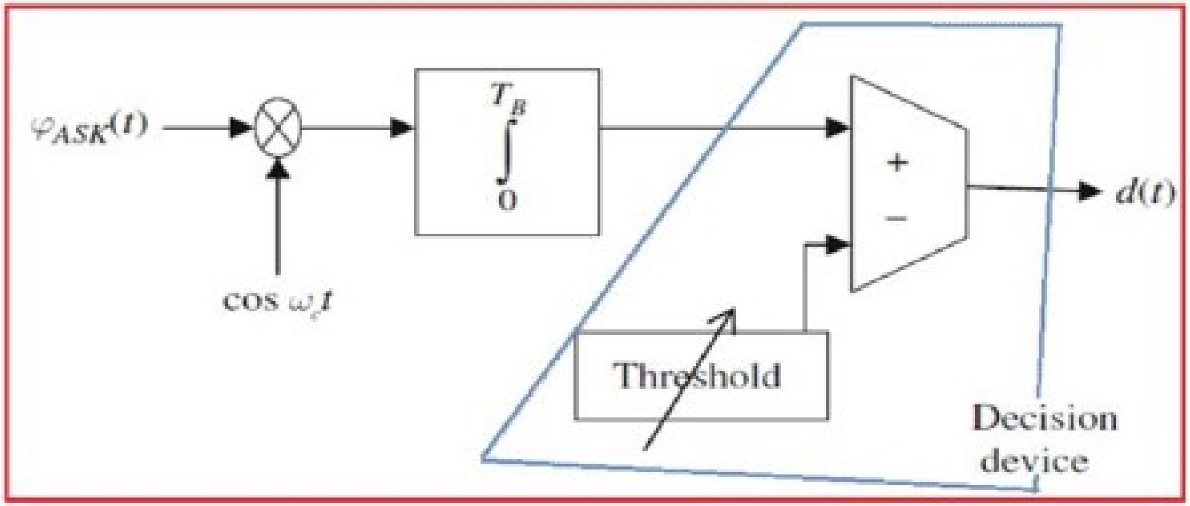


o/p of PM $\Rightarrow$ $s_1(t) \cdot \phi_1(t) \Rightarrow \sqrt{E_b}\, \phi_1^2(t)$

o/p of Integrator $\Rightarrow$ $x_1 = \int_0^{T_b} \sqrt{E_b}\, \phi_1^2(t)\, dt$

ASK Receiver





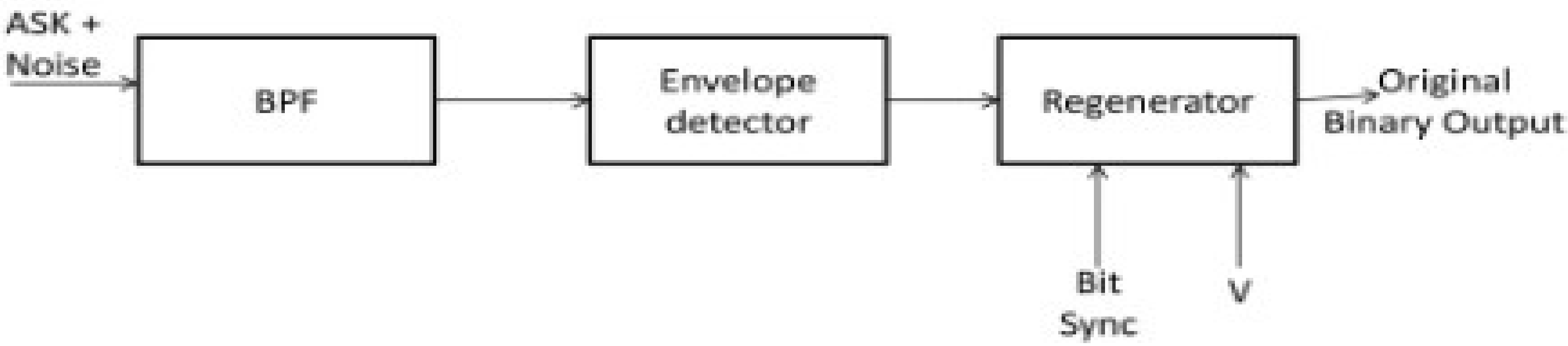
Coherent Detection

- An estimate of the channel phase and attenuation is recovered. It is then possible to reproduce the transmitted signal and demodulate.
- Requires a replica carrier wave of the same frequency and phase at the receiver.
- The received signal and replica carrier are cross-correlated using information contained in their amplitudes and phases.
- Also known as synchronous detection

Non-Coherent Detection

- Requires no reference wave; does not exploit phase reference information (envelope detection)
  - Differential Phase Shift Keying (DPSK)
  - Frequency Shift Keying (FSK)
  - Amplitude Shift Keying (ASK)
  - Non coherent detection is less complex than coherent detection (easier to implement), but has worse performance.

NON-COHERENT DETECTION OF ASK

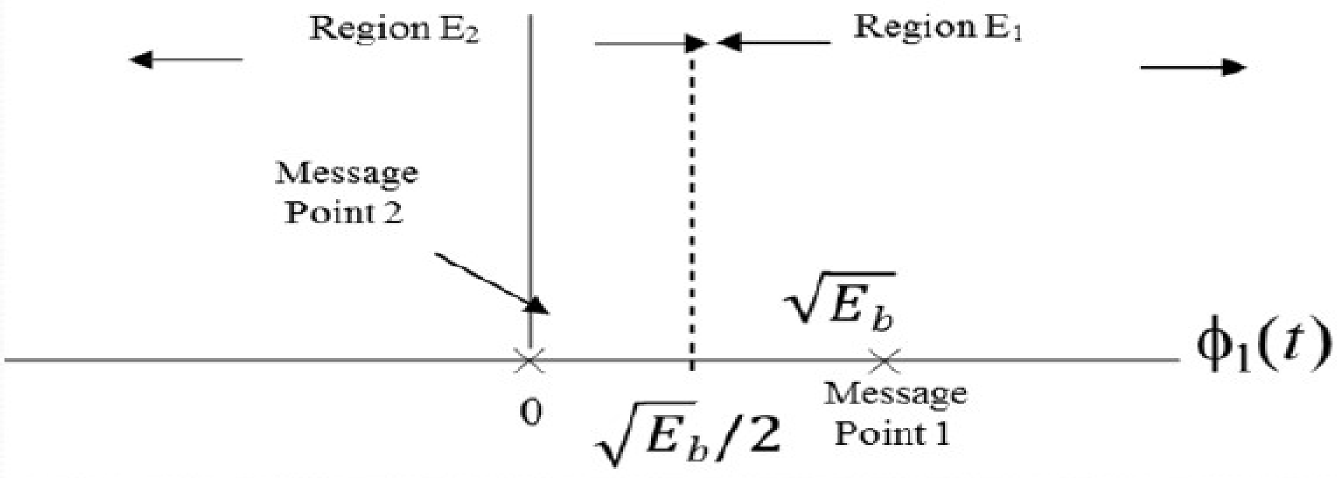


* The BPF passes only carrier signal 'f_c'
* Envelope detector generates high o/p voltage when carrier present.
  " " " low " " " absent.
* The decision device basically regenerative repeater,
  if ED > Threshold ⇒ pulse generated with high voltage & $0 \le t \le T_b$
  if ED < " ⇒ " " zero " & $0 \le t \le T_b$

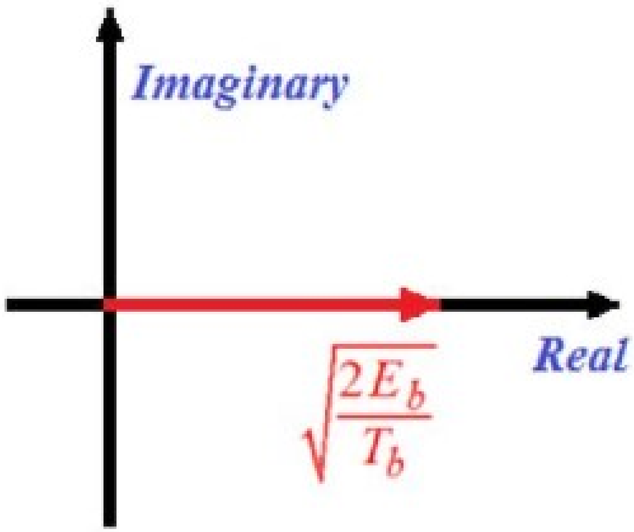
ASK Space diagram

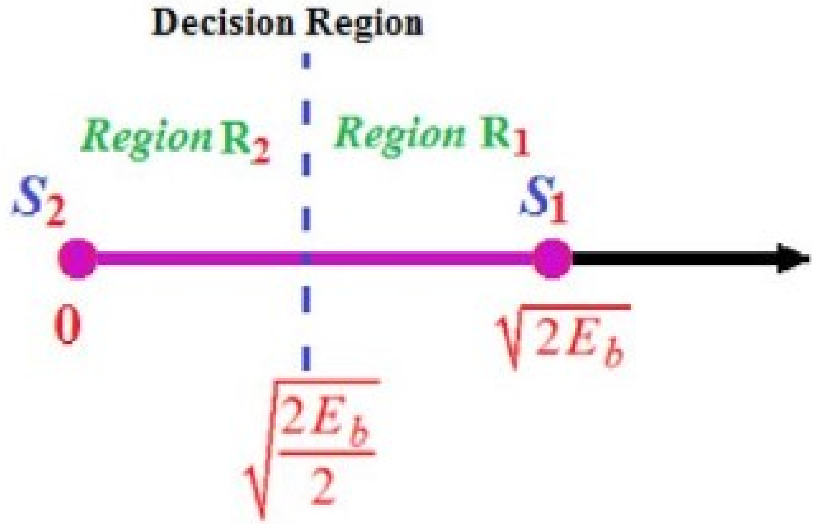
- For binary '1' $s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$
- For binary '0' $s_2(t) = 0$ $0 \le t < T_b$

$$x_j = \int_0^T x(t)\phi_j(t)dt$$
$$= s_{ij} + w_i, \quad j = 1, 2, \ldots, N$$



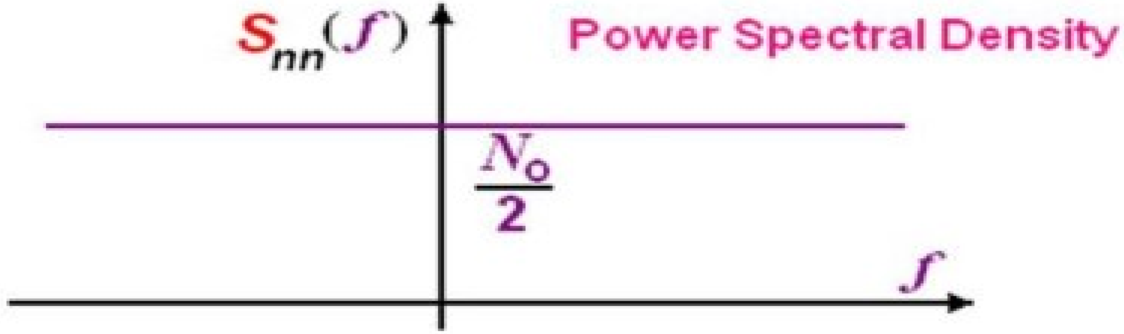
Signal Space: ASK Constellation Diagram





$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t + j\,0 \quad 0 \le t \le T_b$$

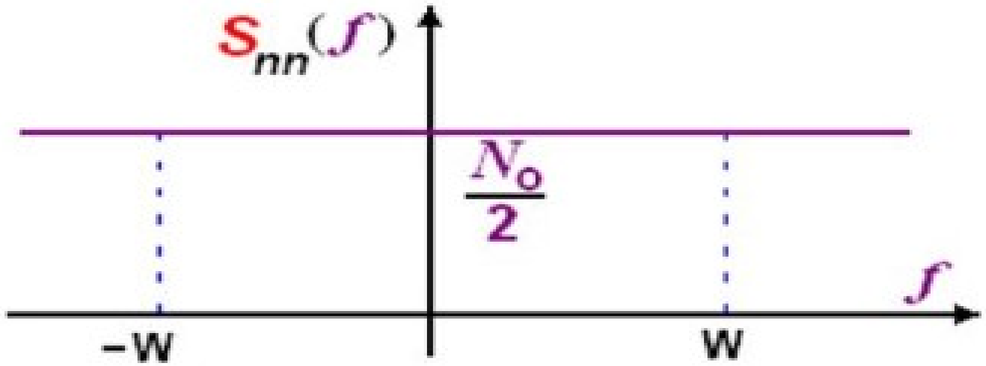
Additive White Noise



Average Power of Additive White Noise

$$P_n = \int_{-W}^{W} S_{nn}(f)\, df$$

$$= \frac{N_0}{2} 2W = N_0 W$$



Noise is assumed to be uncorrelated with the signal

Gaussian Noise

- Let $\eta(t)$ denote a noisy signal.
- The values of noisy signal are ***unpredictable*** and only a probability can be associated to them,

$$\Pr\{n_1 < \eta(t) \le n_2\}$$

- The most common type of noise found in a communication system is ***Gaussian*** with PDF

$$P_{\eta(t)}(n) = \frac{1}{\sqrt{2\pi\sigma_\eta^2}} e^{-\frac{1}{2}\left(\frac{n-\mu_\eta}{\sigma_\eta}\right)^2}$$

where $\mu_\eta = E\{\eta(t)\}$: Mean value , and

$$\sigma_\eta^2 = E\left\{\left(\eta(t) - \mu_\eta\right)^2\right\}$$: Variance

- Probability of error P_e of binary 0

$$P_e = \int_{d_{ik}/2}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(x)^2\right) dx$$

$$P_{e0} = \frac{1}{\sqrt{\pi N_0}} \int_{\sqrt{\frac{E_b}{2}}}^{\infty} \exp\left[-\frac{(x-0)^2}{N_o}\right] dx$$

Let $Z = \frac{x}{\sqrt{N_0}}$

$$P_{e0} = \frac{1}{\pi} \int_{\sqrt{\frac{E_b}{4N_0}}}^{\infty} \exp(-z^2) dz$$

$$P_{e0} = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{4N_0}}\right)$$

Similarly for binary '1'

$$P_{e1} = \frac{1}{2} erfc\left[\sqrt{\frac{E_b}{4N_0}}\right]$$

The total probability of error $= \frac{1}{2}[P_{e0} + P_{e1}]$

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{4N_0}}\right)$$

Also

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\frac{d_{ik}}{2\sqrt{N_0}}\right)$$

Error Performance: Bit Error Rate (BER)

The BASK signal at the receiver in the presence of noise is given by

$$r(t) = \alpha s(t) + n(t)$$

where α is attenuation introduced by Txion channel. Assume α =1.

$n(t)$ is AWGN with power spectral density $\frac{N_0}{2}$ W/Hz.

Then the Bit Error Rate for ASK is given by

$$\text{BER}_{\text{ASK}} = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

where $E_b = \frac{A_c^2 T_b}{4}$, where again E_b is the average energy bit.

Ex: Binary data is transmitted using ASK through a channel that adds white Gaussuan noise with power spectral density $N_0 = 10^{-11}$ W / Hz. Determine the amplitude of a received carrier burst to provide a $BER = 10^{-5}$ for the following data rates

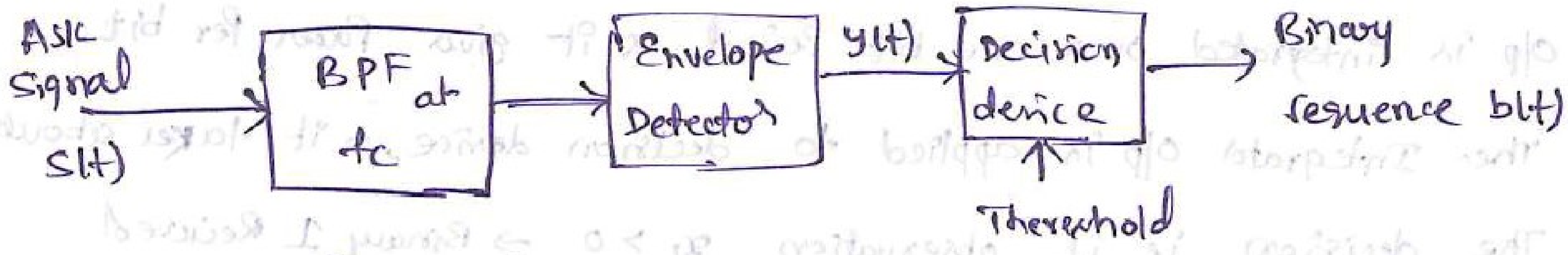
(a) 300 bps; (b) 3kbps; (c) 9.6 kbps

Ans: The Bit Error Rate for ASK is given by $\text{BER}_{\text{ASK}} = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$

To achieve a $BER = 10^{-5}$ $\sqrt{\frac{E_b}{N_0}} = 4.27 \Rightarrow \frac{E_b}{N_0} = 4.27^2 = 18.233$

$N_0 = 10^{-11}$; $E_b = 18.233\text{x}10^{-11}$

(ii) Non Coherent Detection:-



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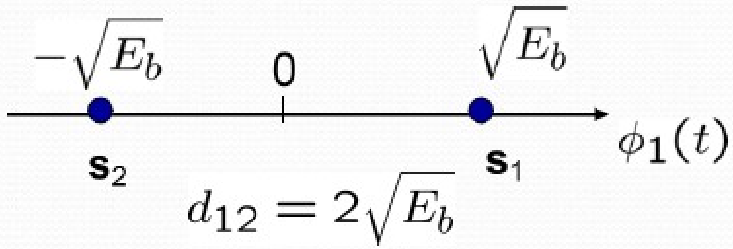
* Envelope detector generates high o/p voltage when carrier present.
  " " " low " " " Absent.

* The decision device basically regenerative repeater, ED o/p
  if ED > Threshold ⇒ pulse generated with high voltage 1 $0 \le t \le T$
  if ED < " ⇒ " " " zero " $0 \le t \le T$

Binary Phase-Shift Keying (BPSK)

- A binary PSK system is therefore characterized by having a signal space that is one-dimensional (i.e. N=1), and with two message points. The pair of signals differ by phase shift of 180^0



C o h e r e n t B i n a r y P S K • In a coherent binary PSK system, the pair of signals, s1(t) and s2(t), used to represent binary symbols 1 and 0, respectively, are defined by

- $S_i(t) = s_{i1}\phi_1(t) \quad for\ i=1,2$
- $S_1(t) = s_{11}\phi_1(t) \quad for\ i=1$
- $S_2(t) = s_{21}\phi_1(t) \quad for\ i=2$

$$s_1(t) = \sqrt{E_b}\phi_1(t)$$

$$s_2(t) = -\sqrt{E_b}\phi_1(t)$$

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t)$$

$$0 \leq t < T_b$$

Binary Phase-Shift Keying (BPSK)

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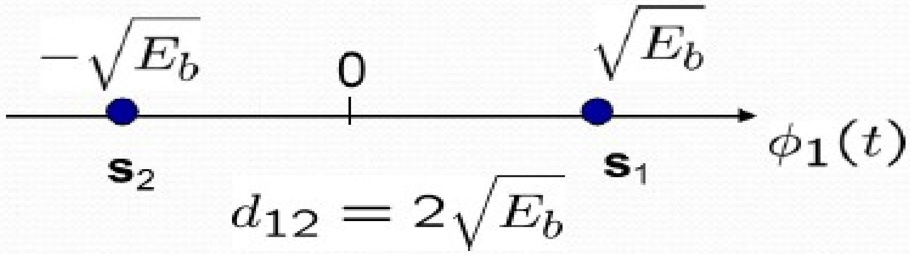


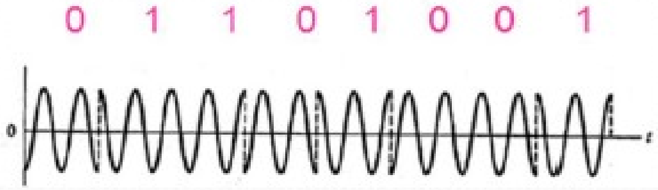
- $S_i(t) = s_{i1}\phi_1(t)$ for $i=1,2$
- $S_1(t) = s_{11}\phi_1(t)$ for $i=1$
- $S_2(t) = s_{21}\phi_1(t)$ for $i=2$

$$s_1(t) = \sqrt{E_b}\phi_1(t)$$

$$s_2(t) = -\sqrt{E_b}\phi_1(t)$$

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t)$$

$$0 \le t < T_b$$



- For Binary '1' $s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$
- For Binary '0' $s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi)$
- T_b is bit duration $s_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$

$0 \le t < T_b$

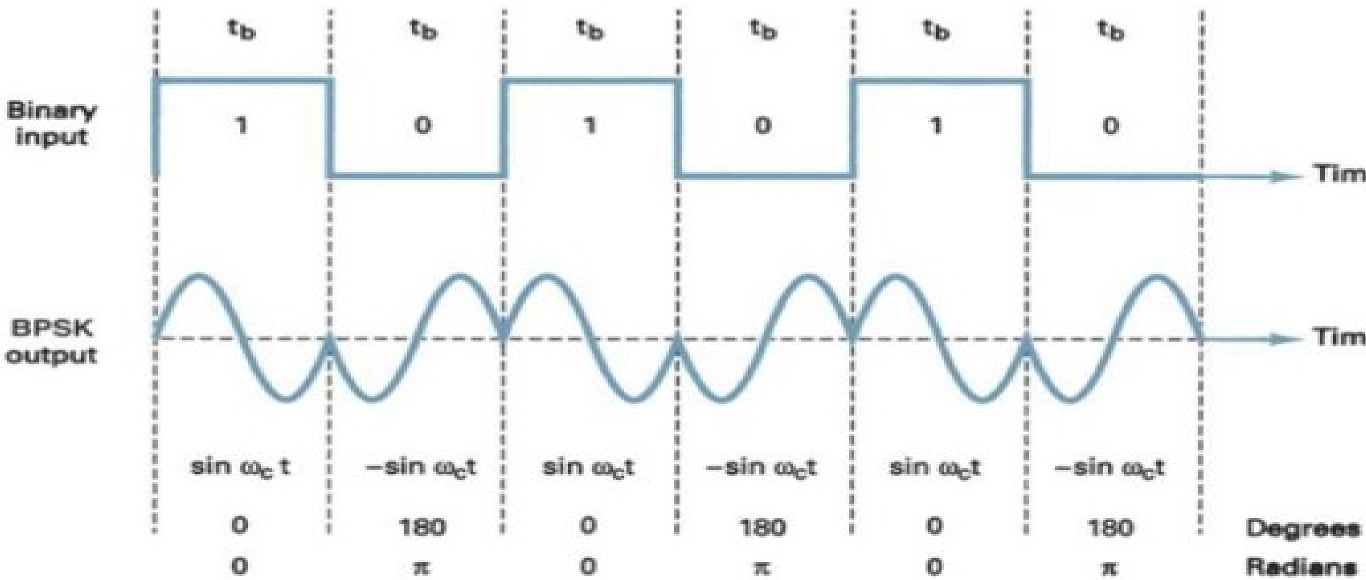
- The pair of signals differ by phase shift of 180^0
- f_c carrier frequency, chosen to be n_c/T_b for some fixed integer n_c
- E_b is transmitted signal energy per bit, $\int_0^{T_b} s_1^2(t)dt = \int_0^{T_b} s_2^2(t)dt = E_b$

BPSK

- Phase-shift keying (PSK) is another form of angle-modulated, constant-amplitude digital modulation.
- BPSK is a form of square-wave modulation of a *continuous wave (CW) signal.*
- Vary the phase shift of the carrier signal to represent digital data.

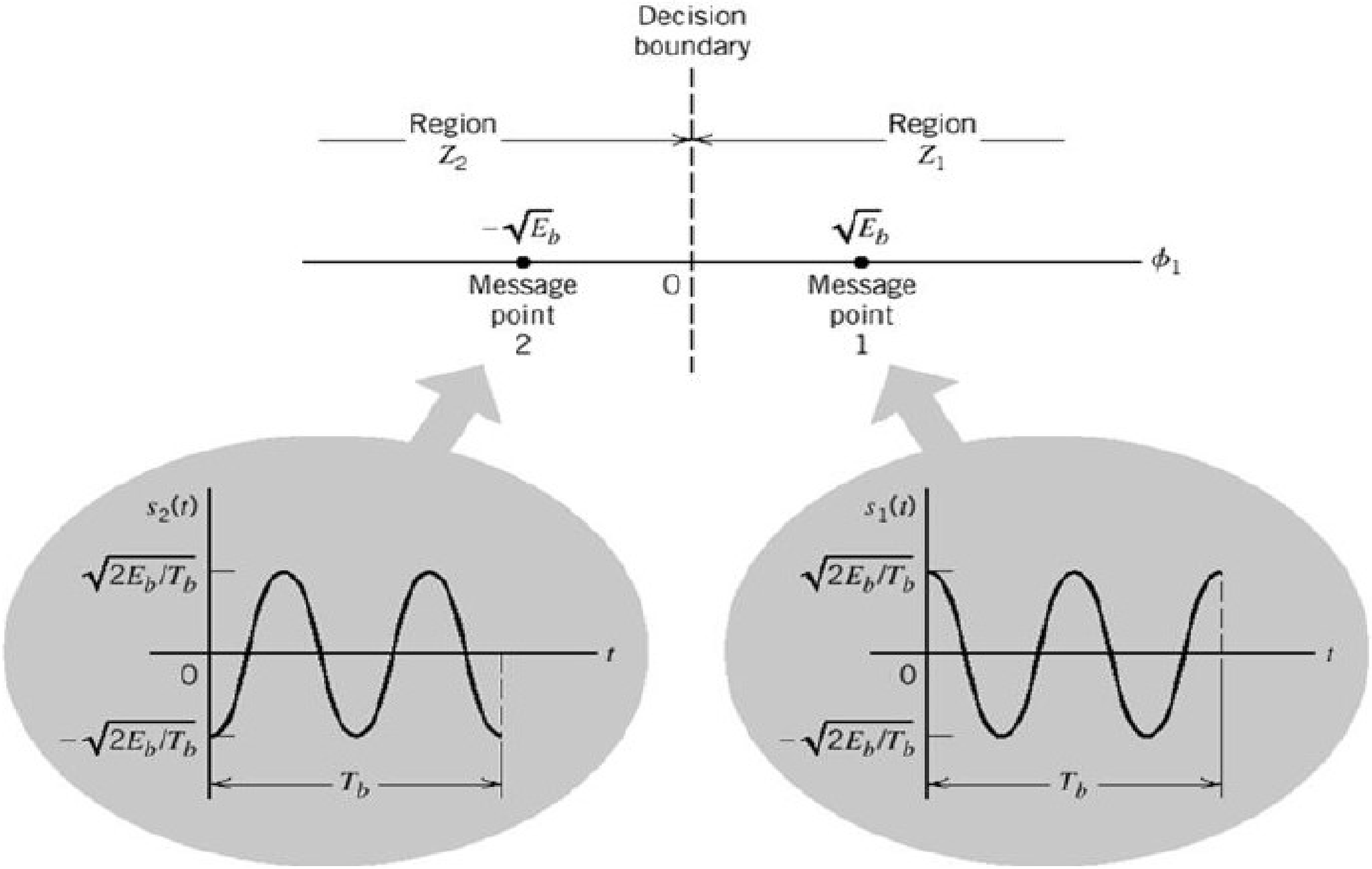
$$s(t)=\begin{cases} s_1(t)=\sqrt{\dfrac{2E_b}{T_b}}\cos 2\pi f_c t, & \text{for Symbol '1'} \\ s_2(t)=\sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t+\pi)=-\sqrt{\dfrac{2E_b}{T_b}}\cos 2\pi f_c t & \text{for Symbol '0'} \end{cases},0\le t\le T_b$$

Digital Modulation

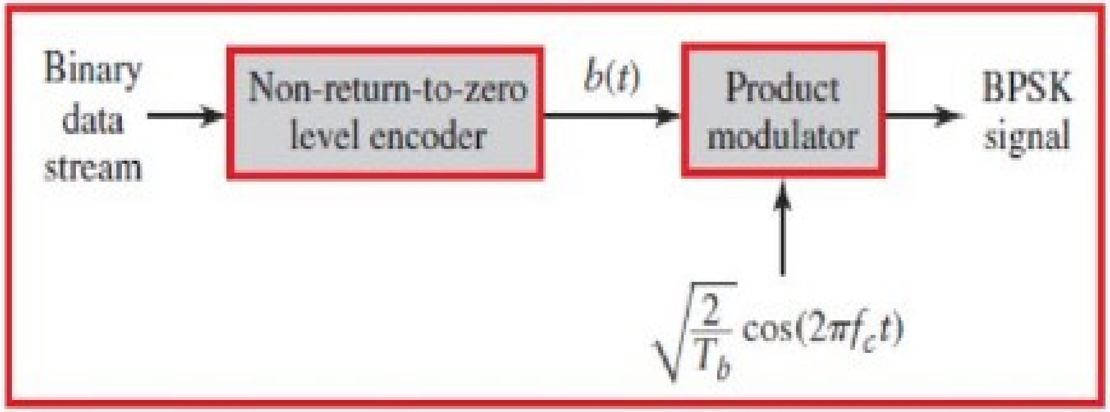


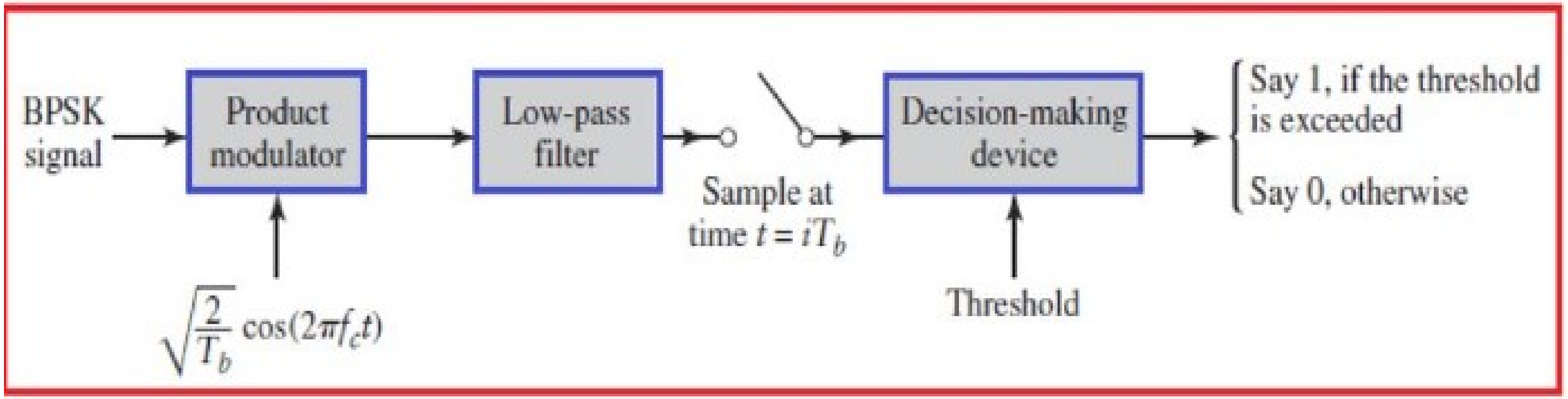
Signal Space Representation for BPSK

- BPSK

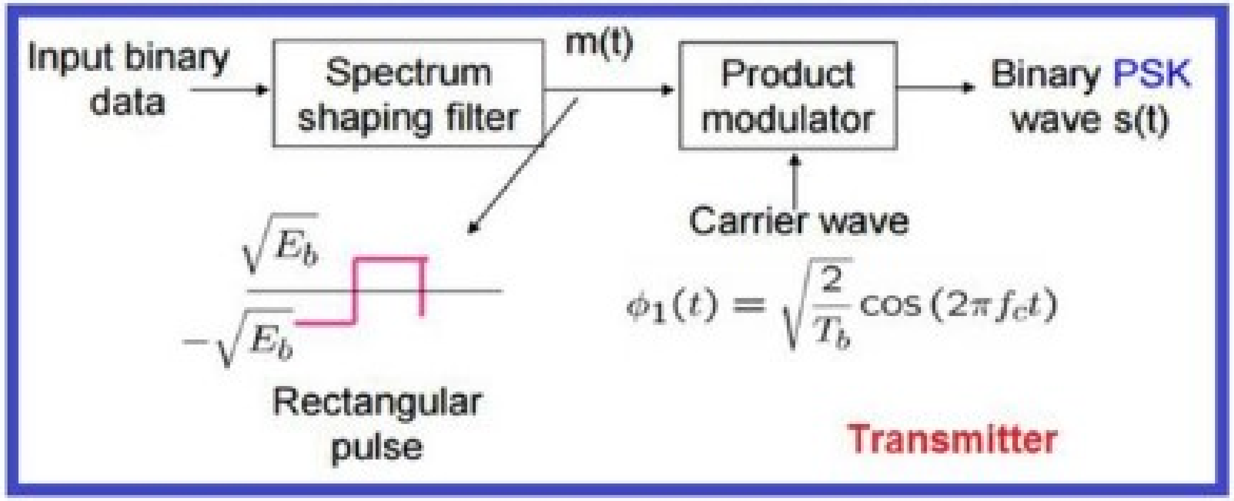


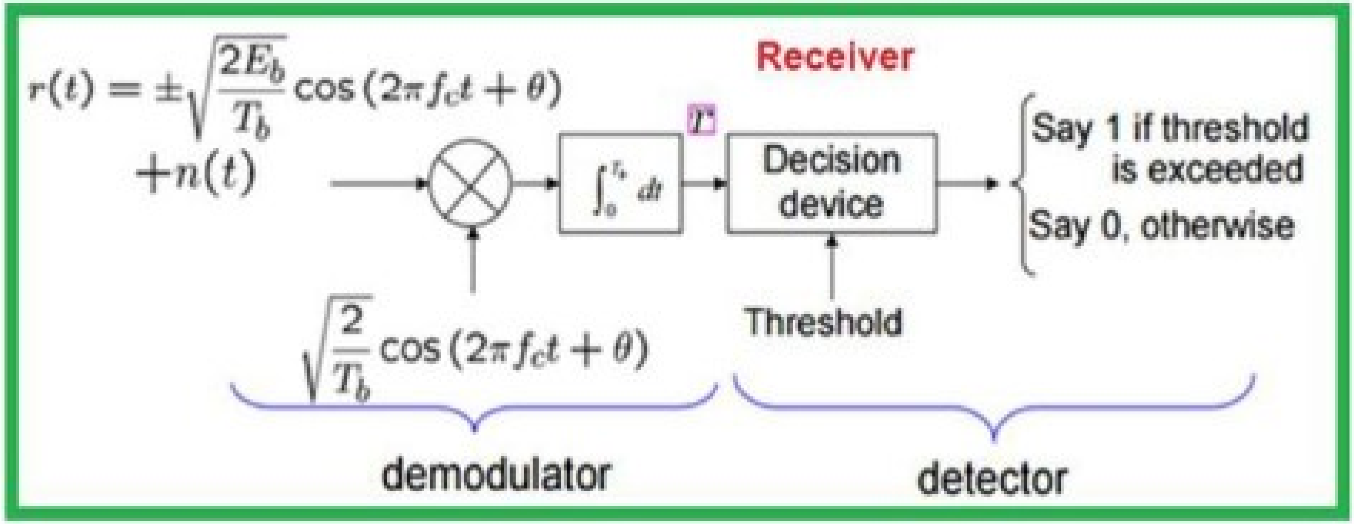
Coherent BPSK Transmitter and Receiver





BPSK Transmitter and Receiver





To generate a binary PSK wave, the input binary sequence in polar form with symbols 1 and 0 represented by constant amplitude levels of $+\sqrt{E_b}$ and $-\sqrt{E_b}$, respectively.

- This binary wave and a sinusoidal carrier were ϕ_1 (t) (whose frequency f_c n_c/T_b for some fixed integer n_c) are applied to a product modulator, as in Figure 4.2b.

- The carrier and the timing pulses used to generate the binary wave are usually extracted from a common master clock. The desired PSK wave is obtained at the modulator output.

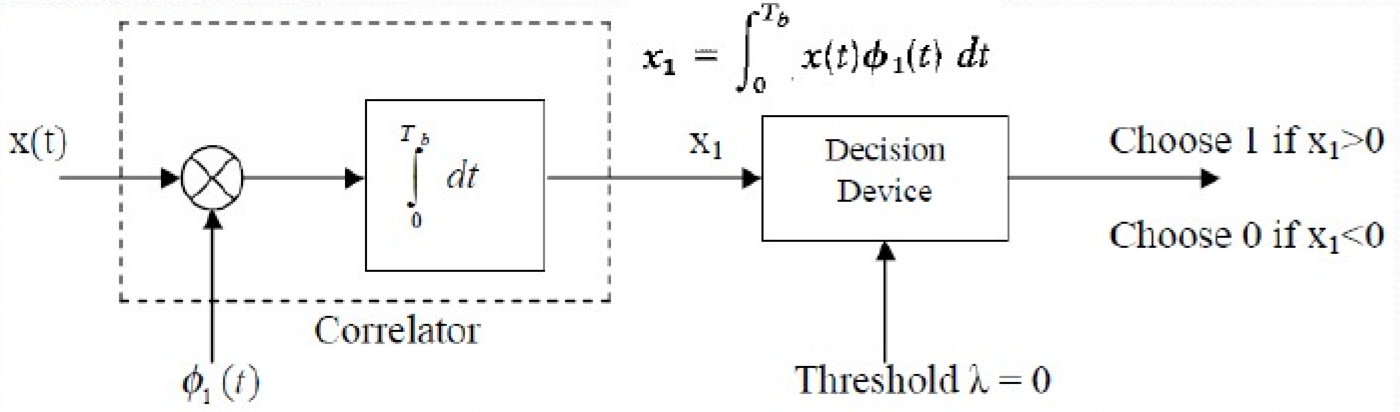
- To detect the original binary sequence of 1s and 0s, we apply the noisy PSK wave $x(t)$ (at the channel output) to a correlator, which is also supplied with a locally generated coherent reference signal $\phi_1(t)$, as in Figure 4.2b.

- The correlator output, x_1, is compared with a threshold of zero volts.

If $x_1 > 0$, the receiver decides in favor of symbol 1.

If $x_1 < 0$, it decides in favor of symbol 0.

BPSK Receiver



A coherent binary PSK system is therefore characterized by having a signal space that is one-dimensional (i.e., N = 1) and with two message points (i.e., M = 2),
The coordinates of the message points equal

$$s_{11} = \int_0^{T_b} s_1(t)\phi_1(t)dt = +\sqrt{E_b}$$

$$s_{21} = \int_0^{T_b} s_2(t)\phi_1(t)dt = -\sqrt{E_b}$$

- The message point corresponding to $s_1(t)$ is located at $s_{11} = +\sqrt{E_b}$, and the message point corresponding to $s_2(t)$ is located at $s_{11} = -\sqrt{E_b}$.

- We must partition the signal space of Fig. 4.1 into two regions:

1. The set of points closest to the message point at. $+\sqrt{E_b}$
2. The set of points closest to the message point at $-\sqrt{E_b}$

- This is accomplished by constructing the midpoint of the line joining these two message points, and then marking off the appropriate decision regions. In Figure 4.1 these decision regions are marked Z_1 and Z_2 according to the message point around which they are constructed.

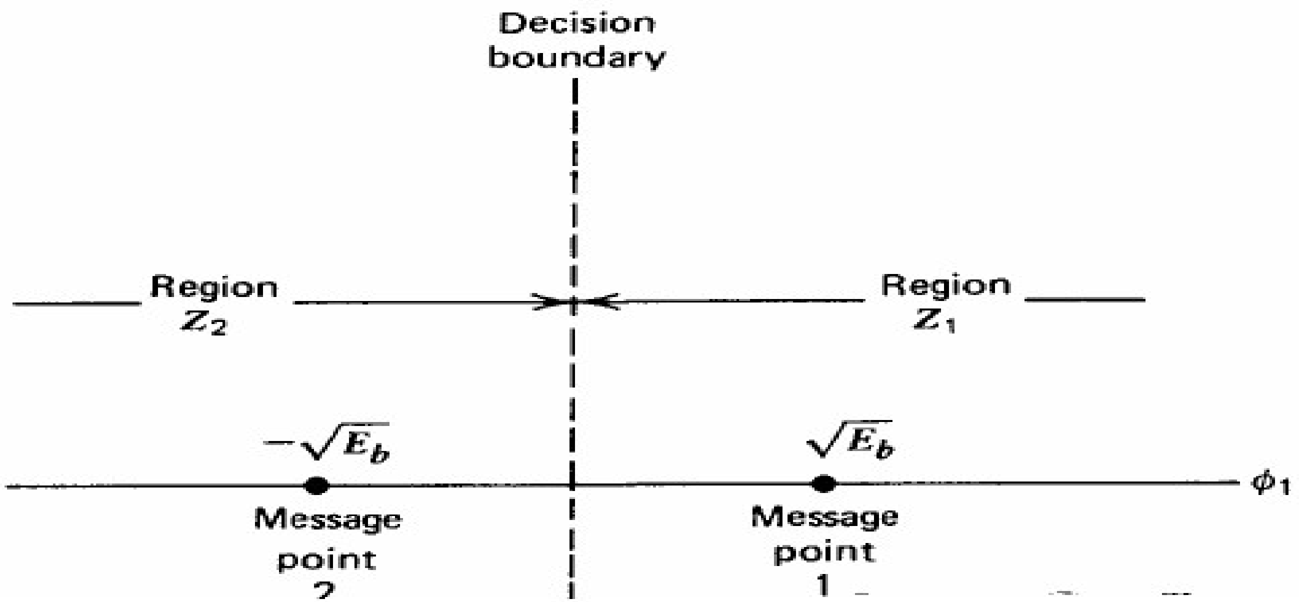


FIGURE 4.1 Signal space diagram for coherent binary PSK system

BPSK probability of error

$$s_{11} = \int_0^{Tb} s_1(t)\phi_1(t)dt = \sqrt{E_b}$$

$$s_{21} = \int_0^{T} s_2(t)\phi_1(t)dt = -\sqrt{E_b}$$

Desicion Regions

$$Z_1: \quad 0 \langle x_1 \langle \infty$$

$$Z_2: -\infty \langle x_1 \langle 0$$

$$x_1 = \int_0^{Tb} x(t)\phi_1(t)dt$$

$$f_{x_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - s_{21})^2\right]$$

$$= \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right]$$

- Probability of error when binary ‘0’ is sent and received as ‘1’

$$P_{e0} = \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{(x_1 + \sqrt{E_b})^2}{N_0} \right] dx_1$$

$$\text{Put } Z = \frac{x_1 + \sqrt{E_b}}{\sqrt{N_0}}$$

$$P_{e0} = \frac{1}{\sqrt{\pi}} \int_{\sqrt{(E_b/N_0)}}^{\infty} \exp\left[(-Z)^2\right] dz$$

$$P_{e0} = \frac{1}{2} erfc \sqrt{\frac{E_b}{N_0}}$$

- Similarly ‘1’ sent and received ‘0’

$$P_{e1} = \frac{1}{2} erfc \sqrt{\frac{E_b}{N_0}}$$

- Total P_e is average of P_{e0} and P_{e1}

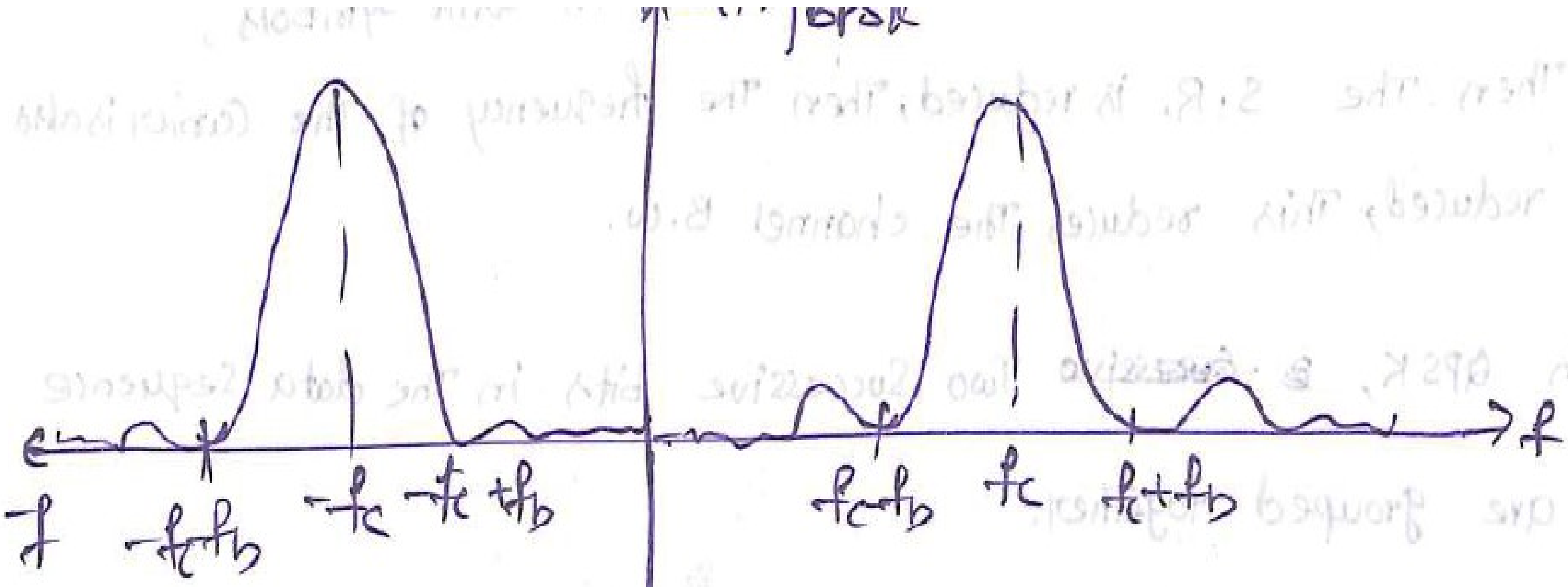
$$P_e = \frac{1}{2} erfc \sqrt{\frac{E_b}{N_0}}$$

- Probability of error calculation. In the case of equally likely ($Pr(m_o)=Pr(m_1)$), we have

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\frac{d_{ik}}{2\sqrt{N_0}}\right)$$

$$= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Band width in BPSK



$$BW = f_c + f_b - f_c + f_b = 2f_b$$

Quadrature Phase-Shift Keying (QPSK)

QPSK → Quadrature Phase Shift Keying

- **Four** different phase states in **one** symbol period
- **Two** bits of information in each symbol

Phase:	0	π/2	π	3π/2 → possible phase values
Symbol:	00	01	11	10

Odd and even bits are separated in data sequence

Odd bits are in one channel(In-phase)

Even bits are in Quadrature Channel

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right]; \quad 0 \le t < T$$

$i = 1, 2, 3, \text{ and } 4$

- T is symbol duration
- E is signal energy per symbol
- There are 4 symbols for i = 1, 2, 3, and 4
- Data sequence 10 11 00 10 00
- Odd bits 11010
- Even Bits 01000

$$s_i(t) = \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right]\sqrt{\frac{2}{T}}\cos(2\pi f_c t) - \sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right]\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$$

$$= \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right]\phi_1(t) - \sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right]\phi_2(t); \quad 0 \le t < T$$

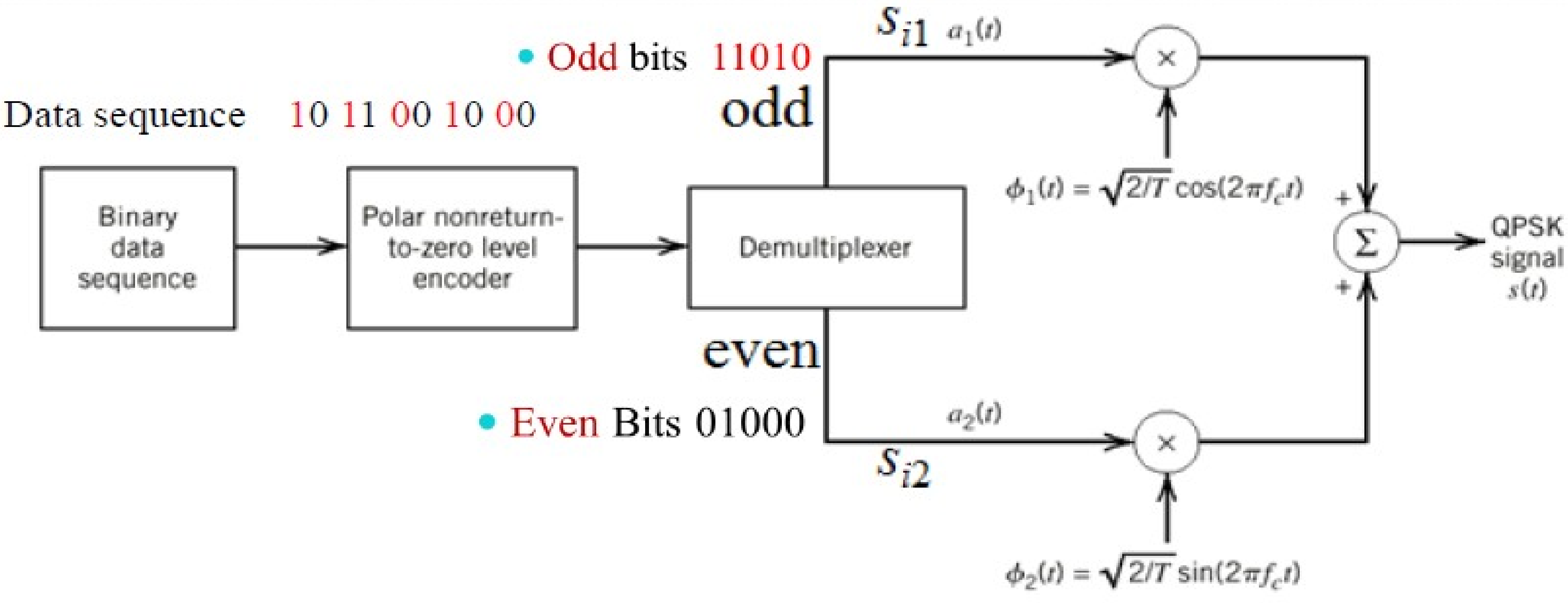
$$\phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_c t), \quad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t), \quad 0 \le t \le T$$

- Which we can write in vector format as

$$\mathbf{s}_i = \begin{bmatrix} \sqrt{E}\cos(2i-1)\frac{\pi}{4} \\ -\sqrt{E}\sin(2i-1)\frac{\pi}{4} \end{bmatrix} \qquad i = 1, 2, 3, \text{ and } 4$$

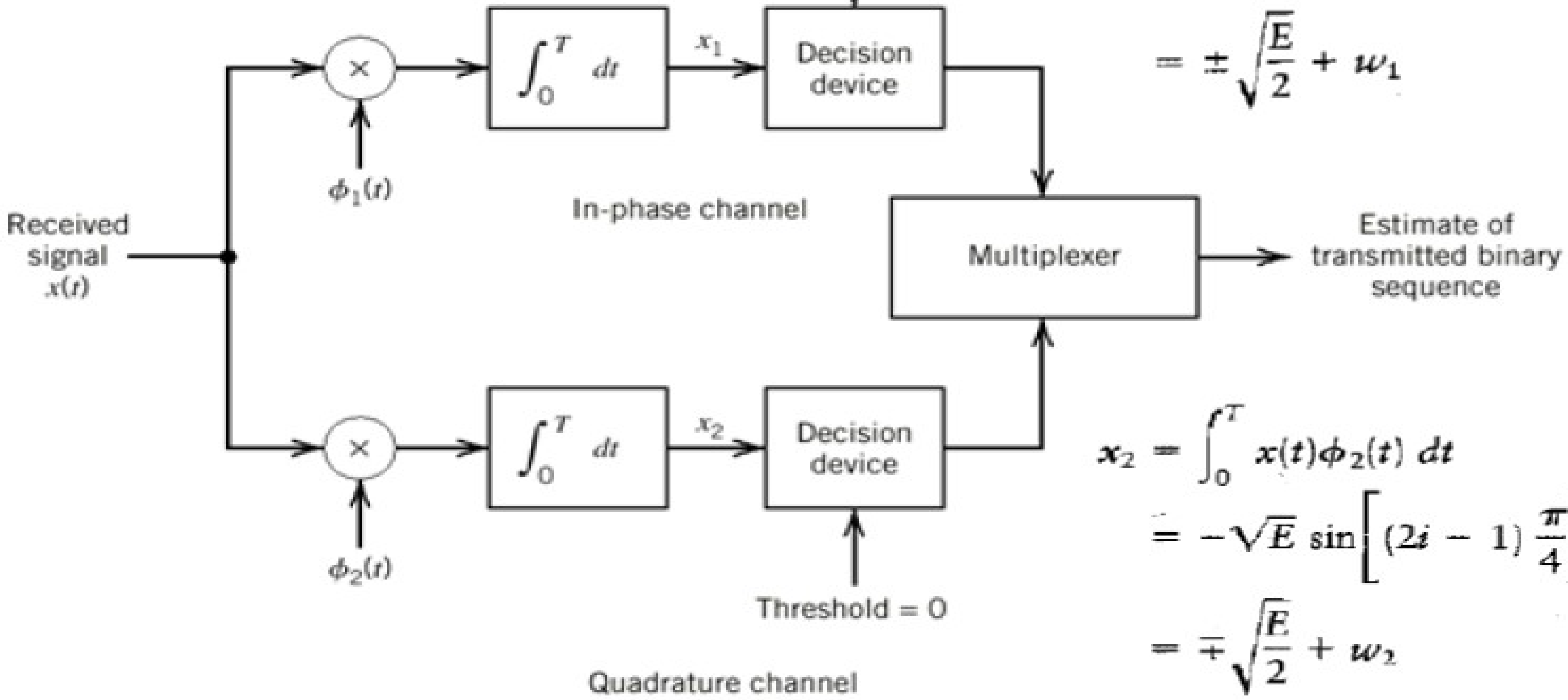
- $s_i(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$



(a)

- Block diagrams of QPSK transmitter

$$x_j = \int_0^T x(t)\phi_j(t)\,dt = s_{ij} + w_i, \quad j = 1, 2, \ldots, N$$



$$x_1 = \int_0^T x(t)\phi_1(t)\,dt = \sqrt{E}\cos\left[(2i-1)\frac{\pi}{4}\right] + w_1 = \pm\sqrt{\frac{E}{2}} + w_1$$

$$x_2 = \int_0^T x(t)\phi_2(t)\,dt = -\sqrt{E}\sin\left[(2i-1)\frac{\pi}{4}\right] + w_2 = \mp\sqrt{\frac{E}{2}} + w_2$$

- Block diagram of Coherent QPSK receiver.

i	Input Dibit	Phase of QPSK signaling	Coordinate of Message point	
			s_{i1}	s_{i2}
1	10	$\pi/4$	$\sqrt{E/2}$	$-\sqrt{E/2}$
2	00	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
3	01	$5\pi/4$	$-\sqrt{E/2}$	$\sqrt{E/2}$
4	11	$7\pi/4$	$\sqrt{E/2}$	$\sqrt{E/2}$

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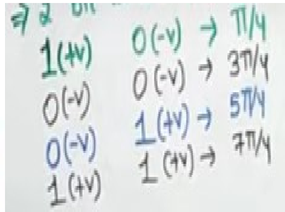
$$VQPSK(t) = b_o(t).\sqrt{P_s}\cos 2\pi f c t + b_e(t).\sqrt{P_s}\sin 2\pi f c t$$



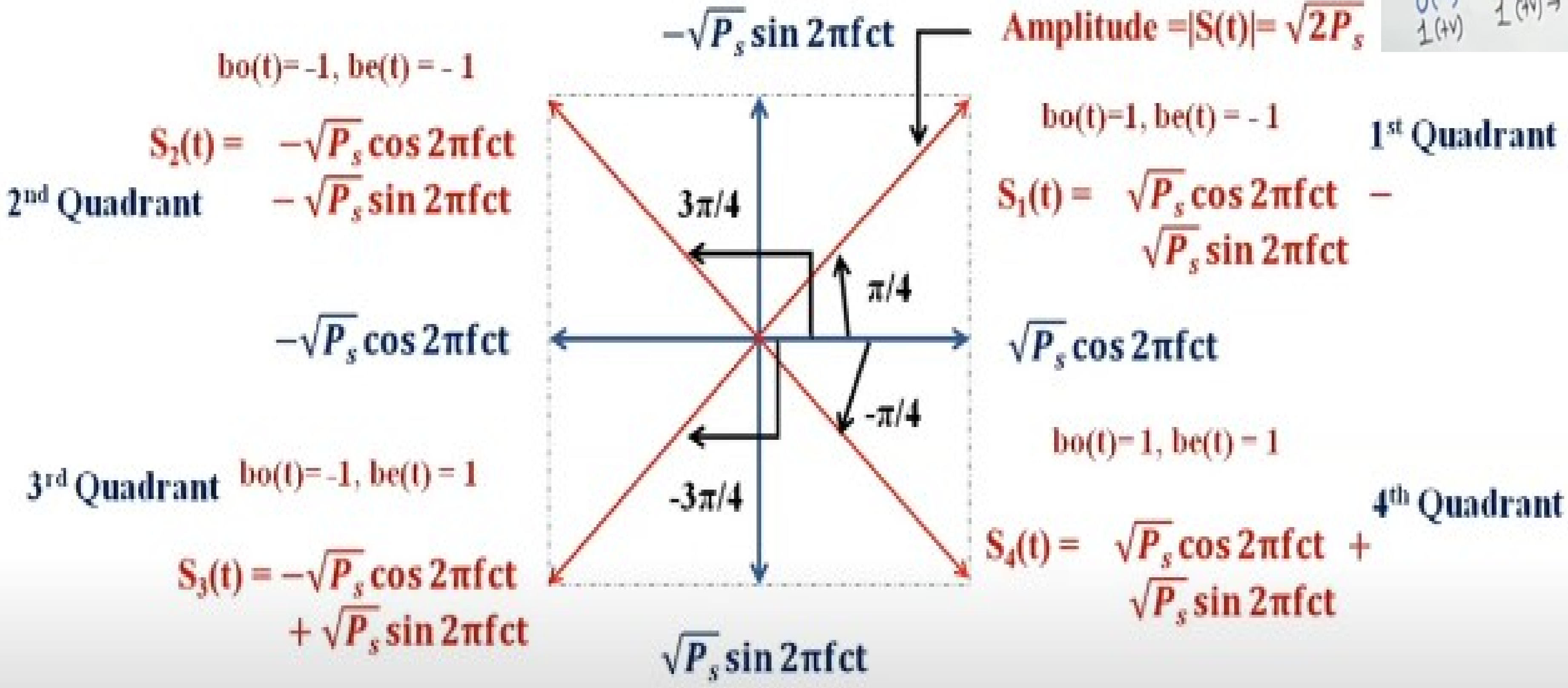
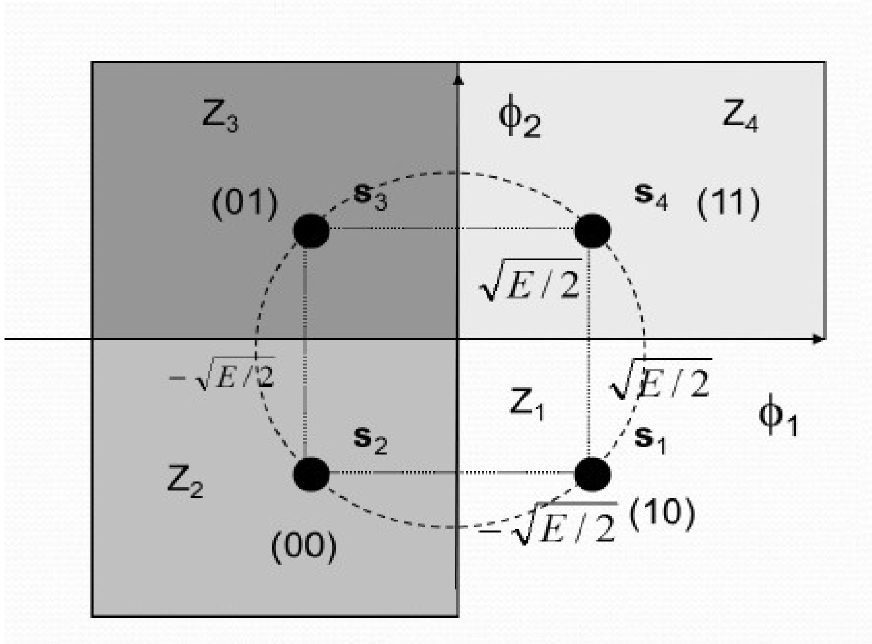
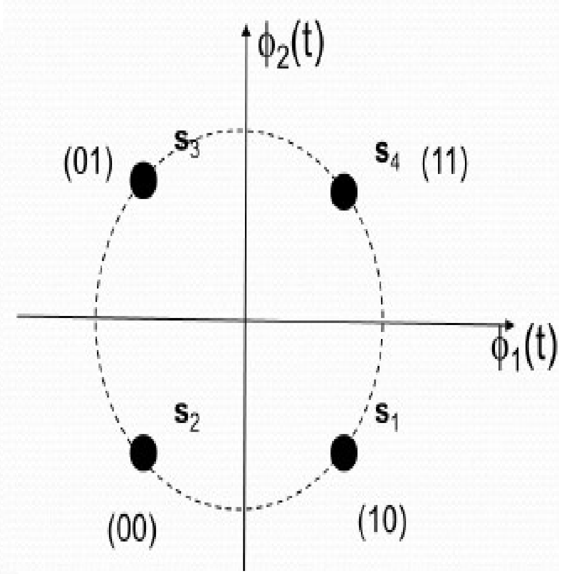


Fig. 3: Phasor Diagram of π/4-QPSK Signal

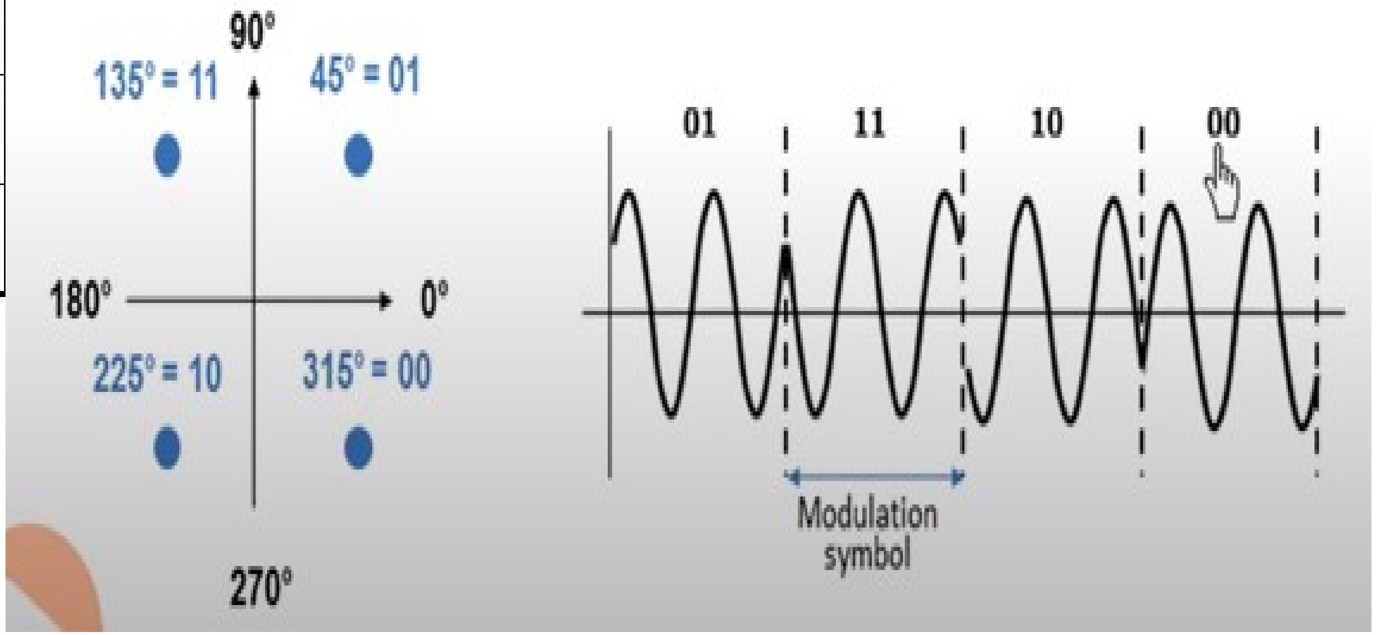
QPSK Constellation

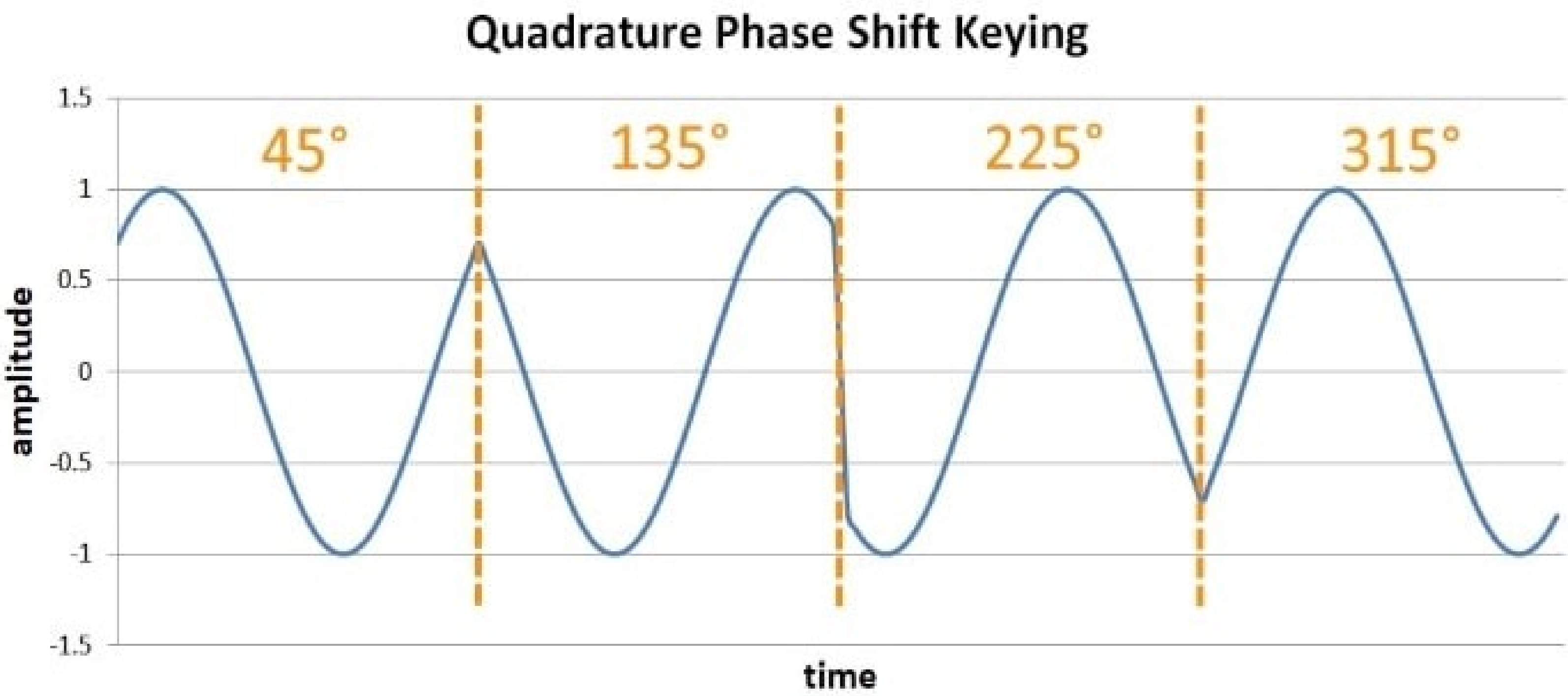




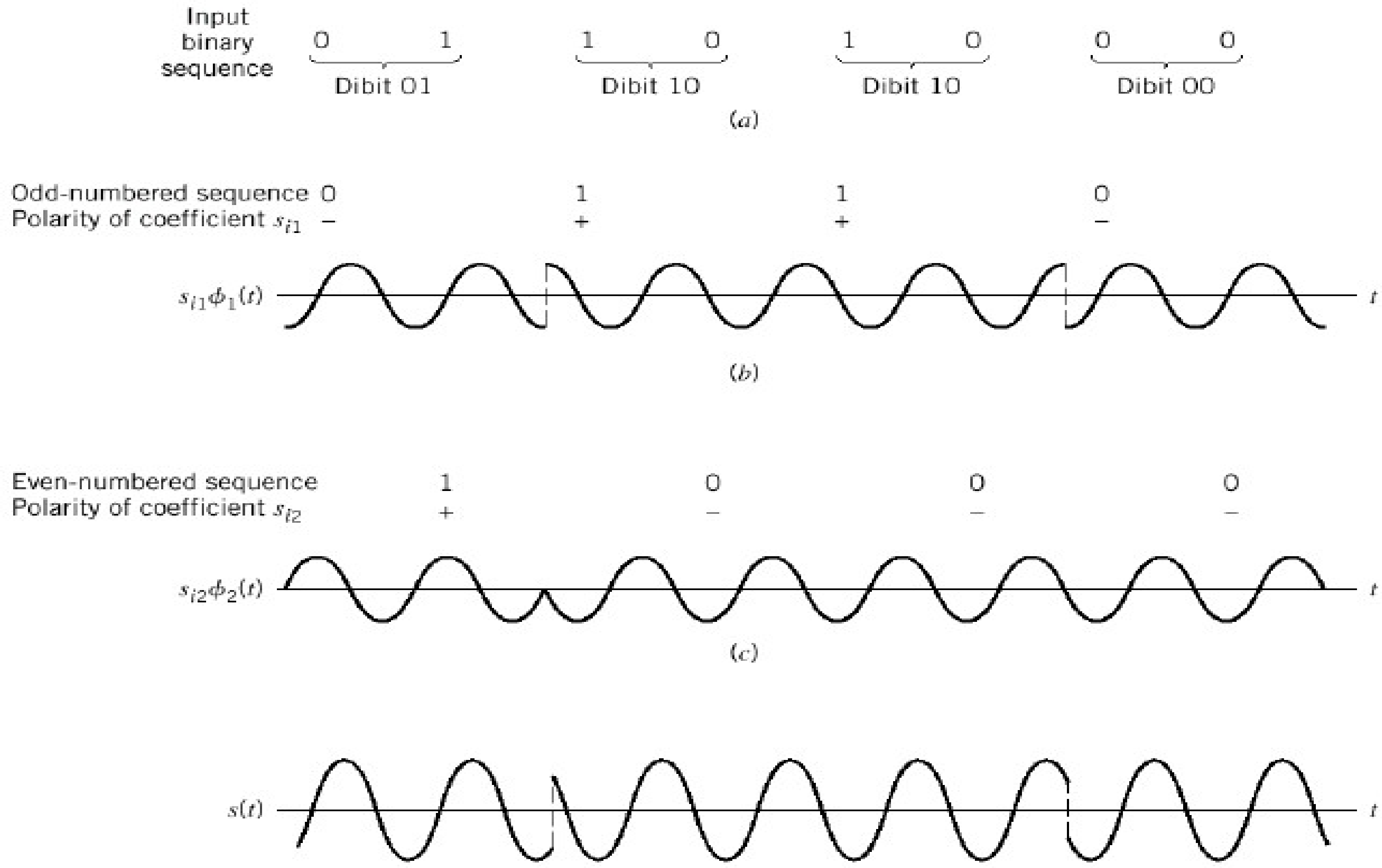
s_{i1}	s_{i2}
$\sqrt{E/2}$	$-\sqrt{E/2}$
$-\sqrt{E/2}$	$-\sqrt{E/2}$
$-\sqrt{E/2}$	$\sqrt{E/2}$
$\sqrt{E/2}$	$\sqrt{E/2}$

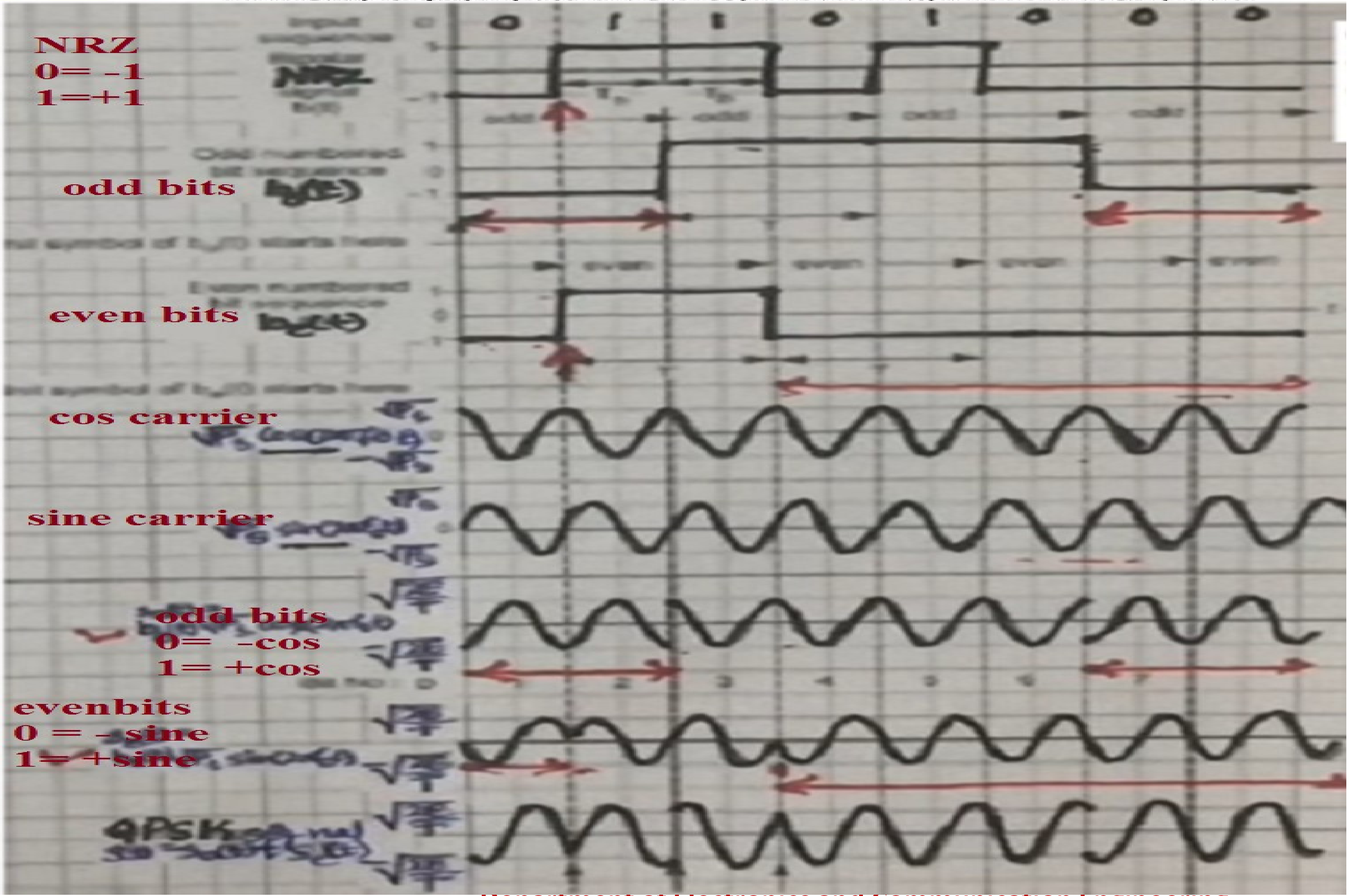
10	$\pi/4$
00	$3\pi/4$
01	$5\pi/4$
11	$7\pi/4$




Quadrature Phase Shift Keying
45°
135°
225°
315°
1.5
1
0.5
0
-0.5
-1
-1.5
amplitude
time

QPSK signals





NRZ
0= -1
1=+1
odd bits
even bits
cos carrier
sine carrier
odd bits
0= -cos
1= +cos
evenbits
0 = -sine
1= +sine

- Bandwidth of BPSK signal, $BW = 2f_b$.

  where f_b is input bit rate and $T_b = \frac{1}{f_b}$, is one bit duration.

- In QPSK, two waveforms forms the baseband signal, In-phase signal $b_o(t)$ and Quadrature signal $b_e(t)$.
- One bit period for these signal is $T_s = 2\,T_b$.
- Bandwidth of QPSK signal is

$$BW = \frac{2}{T_s} = \frac{2}{2Tb} = \frac{1}{T_b} = f_b$$

- we can make the following conclusions:

1. The carrier phase changes by ±180° whenever ***both the in-phase and the quadrature components*** of the QPSK signal change sign (01 to 10)
2. The carrier phase changes by ±90° degrees whenever the ***in-phase or quadrature component*** changes sign (10 to 00 – in-phase changes, quadrature doesn't changes)
3. The carrier phase is unchanged when neither the in-phase nor the quadrature component change sign. (10 and then 10 again).

Conclusion: Situation 1 is of concern when the QPSK signal is filtered during transmission because *the 180 or also 90 degrees shifts in carrier phase might result in changes in amplitude* (envelope of QPSK), which will cause symbol errors

- Whenever both bits are changed simultaneously, 180^0 phase-shift occurs.
- At 180^0 phase-shift, the amplitude of the transmitted signal changes very rapidly costing amplitude fluctuation.
- This signal may be distorted when is passed through the filter or nonlinear amplifier.

Conclusions

- *QPSK is capable of transmitting data twice as faster as BPSK with the same energy per bit.*
- *QPSK has half of the bandwidth of BPSK.*
- *Since one symbol of QPSK consists of two bits, we have* $E = 2E_b$.
- *Symbol duration* $T = 2T_b$.
- *Used in Wireless Communications*

QPSK Probability of error

- We can treat QPSK as the combination of 2 independent BPSK over the interval $T=2T_b$
- since the first bit is transmitted by ϕ_1 and the second bit is transmitted by ϕ_2.
- Probability of error for each channel is given by

$$P' = \frac{1}{2}\operatorname{erfc}\left(\frac{d_{12}}{2\sqrt{N_0}}\right) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$$

- If symbol is to be received correctly both bits must be received correctly.
- Hence, the average probability of correct decision is given by $P_c = (1 - P')^2$
- Which gives the probability of errors equal to

$$P_e = 1 - P_C = \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) - \frac{1}{4}\operatorname{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right)$$

$$\approx \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$$

- Since one symbol of QPSK consists of two bits, we have $E = 2E_b$.

$$Pe(\text{per symbol}) \approx \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

- The above probability is the error probability per symbol. The avg. probability of error per bit

$$Pe(\text{per bit}) = \frac{1}{2} Pe(\text{per symbol}) \approx \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

- Which is exactly the same as BPSK .

M-array PSK

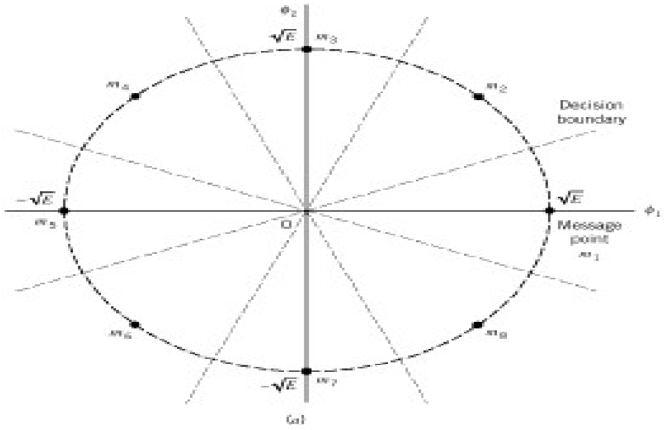
- At a moment, there are M possible symbol values being sent for M different phase values,

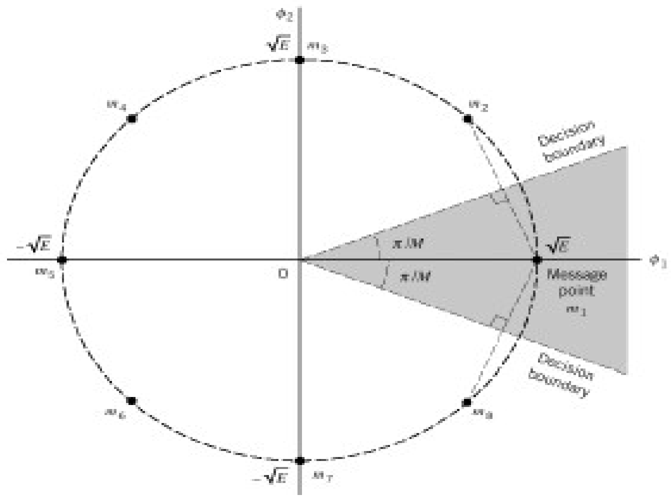
$$\theta_i = 2(i-1)\pi / M$$

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{2\pi}{M}(i-1)\right), \qquad i = 1,2,\ldots,M$$

M-array PSK

- Signal-space diagram for octaphase-shift keying (i.e., $M = 8$). The decision boundaries are shown as dashed lines.
- Signal-space diagram illustrating the application of the union bound for octaphase-shift keying.





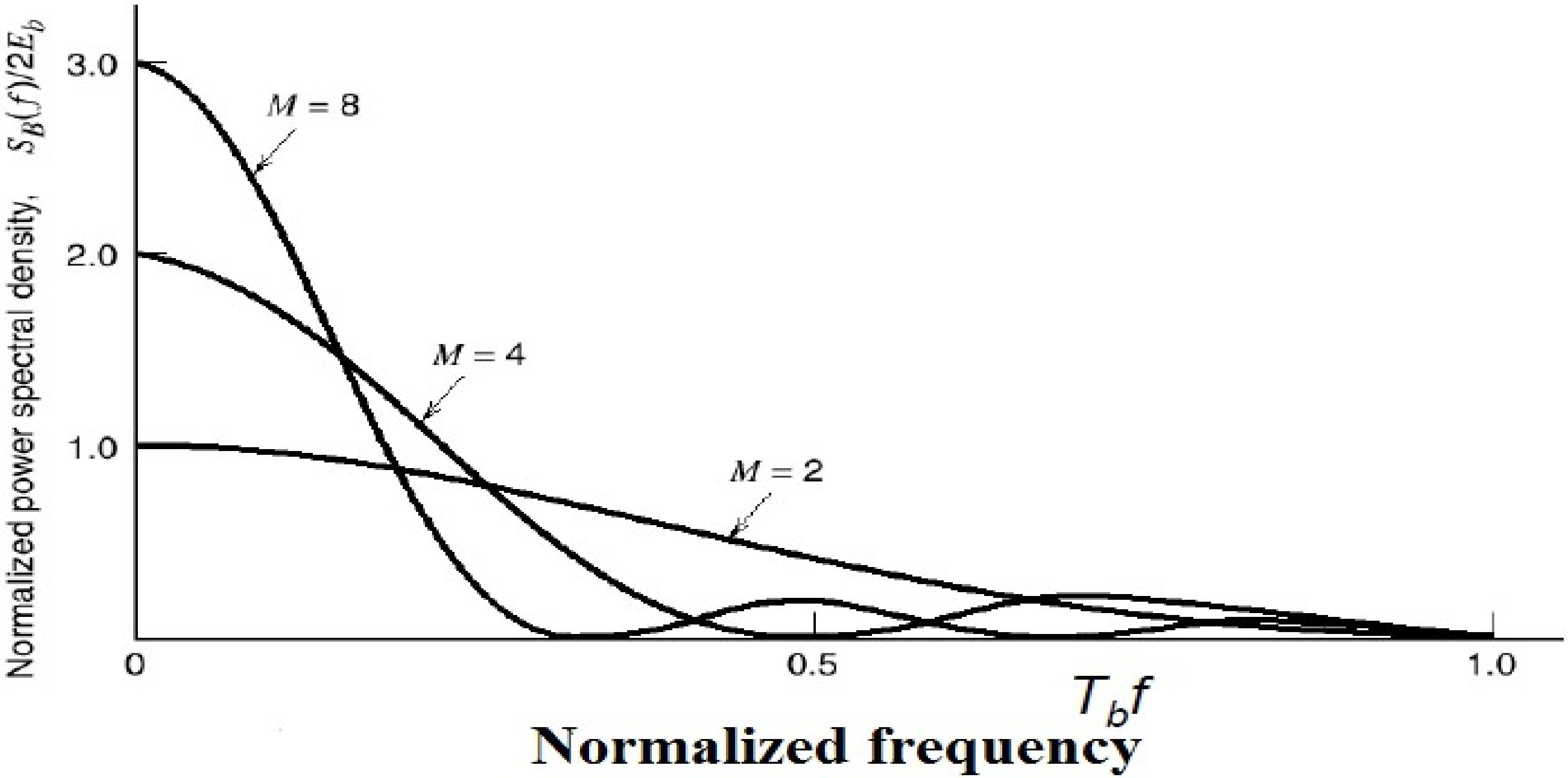
- Bandwidth efficiency:
  - We only consider the bandwidth of the main lobe (or null-to-null bandwidth)

$$B = \frac{2}{T} = \frac{2}{T_b \log_2 M} = \frac{2R_b}{\log_2 M}$$

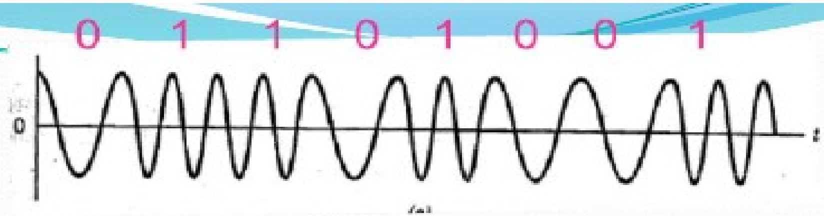
  - Bandwidth efficiency of M-ary PSK is given by

$$\rho = \frac{R_b}{B} = \frac{R_b}{2R_b} \log_2 M = 0.5 \log_2 M$$

- Power spectra of M-ary PSK signals for M = 2, 4, 8.



Binary FSK



- Transmitted signals are

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_i t), & 0 < t \le T_b \\ 0, & \text{elsewhere} \end{cases}$$

- Where

$$f_i = \frac{n_c + i}{T_b}; \quad i = 1,2$$

- Two binary values represented by two different frequencies which are orthogonal to each other
- The signal points $S_1(t)$ and $S_2(t)$ can be expressed as:
- $S_i(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$ for $i=1,2$ $\qquad 0 \le t \le T$

where $\phi_1(t), \phi_2(t)$ *orthonormal basis functions*

- Here two orthonormal basis functions are required to represent $s_1(t)$ and $s_2(t)$. They are written as *for i=1,2*

$$\phi_i(t) = \begin{cases} \sqrt{\dfrac{2}{T_b}} \cos(2\pi f_i t), & 0 < t \le T_b \\ 0, & \text{elsewhere} \end{cases}$$

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_1 t)$$

$$\phi_2(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t)$$

- As a result, the signal vectors are
- $S_i(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$ *for i=1,2*
- $S_1(t)$ represented symbol "1".
- $S_2(t)$ represented symbol "0".

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t)$$

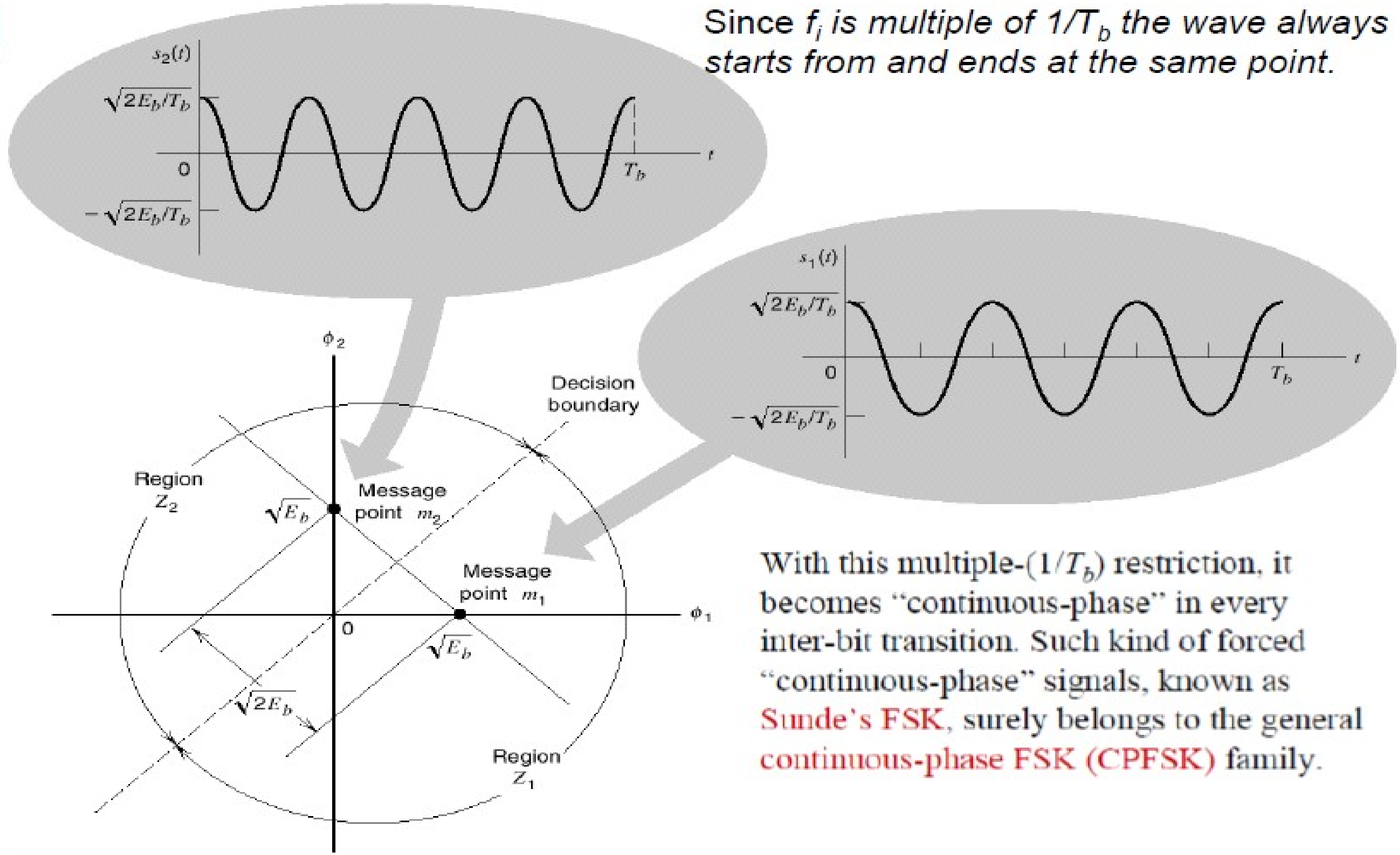
$$s_1(t) = \sqrt{E_b}\phi_1(t)$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t)$$

$$s_2(t) = \sqrt{E_b}\phi_2(t)$$

$$\mathbf{s}_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix} \text{ and } \mathbf{s}_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$$

Since f_i is multiple of $1/T_b$ the wave always starts from and ends at the same point.



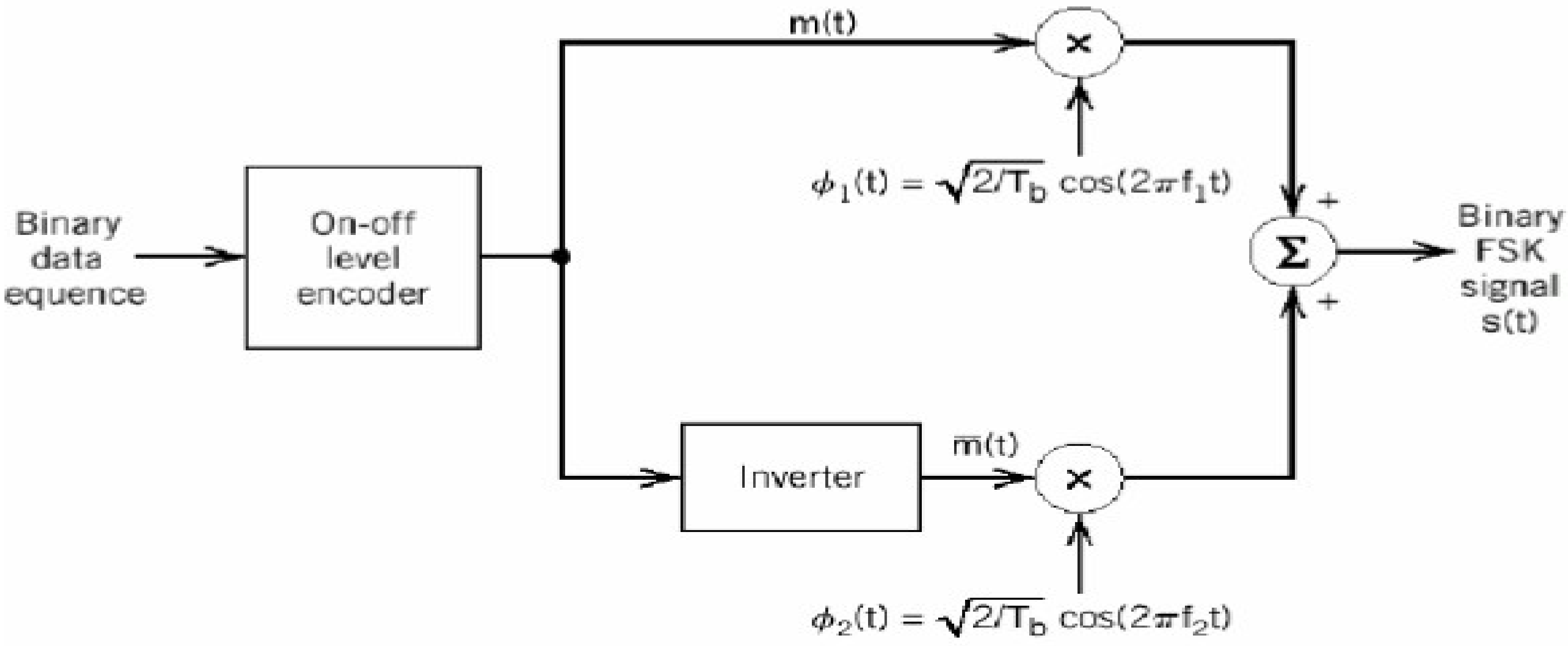
With this multiple-$(1/T_b)$ restriction, it becomes "continuous-phase" in every inter-bit transition. Such kind of forced "continuous-phase" signals, known as Sunde's FSK, surely belongs to the general continuous-phase FSK (CPFSK) family.

Binary FSK Transmitter

f_i : transmitted frequency with separation $\Delta f = f_1 - f_0$

Δf is selected so that s1(t) and s2(t) are orthogonal to each other

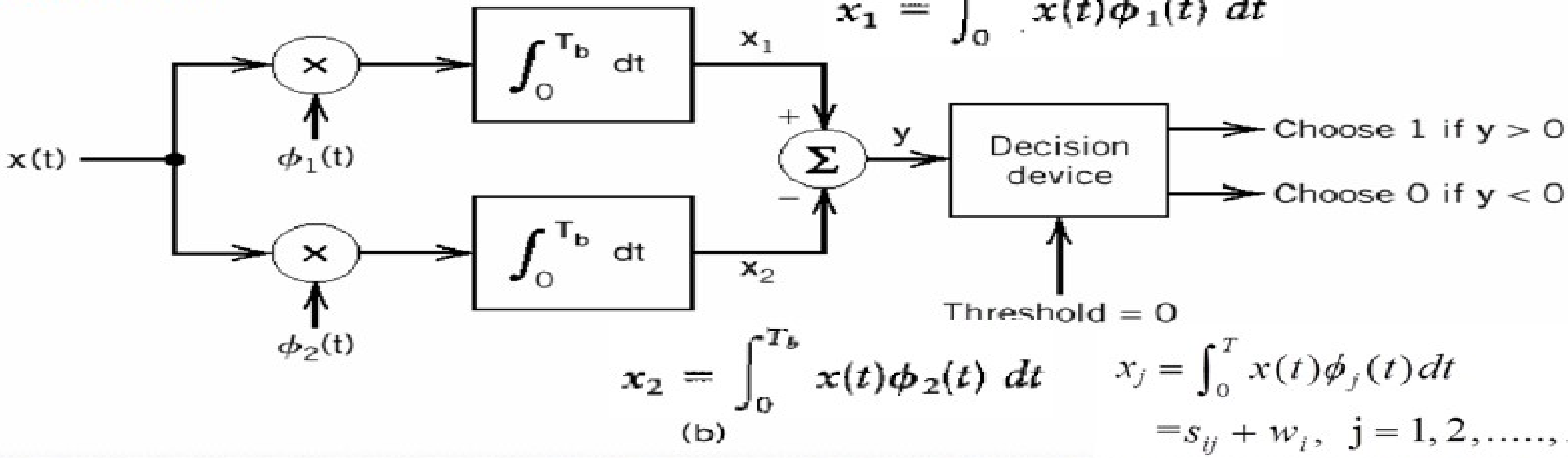
$$\int_0^{T_b} s_1(t)s_2(t)dt = 0$$



Coherent Binary FSK receiver

- Receiver

$$x_1 = \int_0^{T_b} x(t)\phi_1(t)\,dt$$



$$x_2 = \int_0^{T_b} x(t)\phi_2(t)\,dt$$

$$x_j = \int_0^{T} x(t)\phi_j(t)\,dt = s_{ij} + w_i, \quad j = 1, 2, \ldots, N$$

The output $y = x_1 - x_2$, The decision boundary is characterized by the line with $x_1 - x_2 = 0$

If $x_1 - x2 > 0$(the received signal point lies in region Z_1) , the decision device makes a decision of 1)

If $x_1 - x_2 < 0$ (the received signal point lies in region Z_2) ,the decision device makes a decision of 0.)

- If the signal (s_1) was transmitted, the outputs x_1 and x_2 of the correlators can be expressed as:

$$x_1 = \int_0^T \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_1 t) dt = \sqrt{E_b}$$

$$x_2 = \int_0^T \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t) dt = 0.$$

- If the signal (s_2) was transmitted, the outputs of the correlators can be expressed as:

$$x_1 = \int_0^T \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_1 t) dt = 0$$

$$x_2 = \int_0^T \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t) dt = \sqrt{E_b}.$$

BFSK Probability of error

- Gaussian random variable Y whose sample value y
- is equal to the difference between x_1 and x_2; i.e., $y = x_1 - x_2$
- The mean value of the random variable Y depends on which binary symbol was transmitted.
- Given that symbol 1 was sent, the Gaussian random variables X_1 and X_2, whose sample values are denoted by x_1 and x_2, have mean values equal to $\sqrt{E_b}$ and zero, respectively.

Correspondingly, the conditional mean of the random variable Y given that symbol 1 was sent is

$$\mathbb{E}[Y|1] = \mathbb{E}[X_1|1] - \mathbb{E}[X_2|1] = +\sqrt{E_b}$$

- Given that symbol 0 was sent, the Gaussian random variables X_1 and X_2, whose sample values are denoted by x_1 and x_2, have mean values equal to zero and $\sqrt{E_b}$, respectively. Correspondingly, the conditional mean of the random variable Y given that symbol 0 was sent is

$$\mathbb{E}[Y|0] = \mathbb{E}[X_1|0] - \mathbb{E}[X_2|0] = -\sqrt{E_b}$$

- The variance of the random variable Y is independent of which binary symbol was sent.
- Since the random variables X_1 and X_2 are statistically independent, each with a variance equal to $N_0/2$, it follows that

$$\text{var}[Y] = \text{var}[X_1] + \text{var}[X_2] = N_0$$

- Suppose we know that symbol 0 was sent. The conditional probability density function of the random variable Y is then given by

$$f_Y(y|0) = \frac{1}{\sqrt{2\pi N_0}} \exp\left[-\frac{(y+\sqrt{E_b})^2}{2N_0}\right]$$

- Since the condition $x_1 > x_2$ or, equivalently, $y > 0$ corresponds to the receiver making a decision in favor of symbol 1, we deduce that the conditional probability of error given that symbol 0 was sent is

$$p_{10} = \mathbb{P}(y > 0 | \text{symbol 0 was sent}) = \int_0^\infty f_Y(y|0)\, dy$$

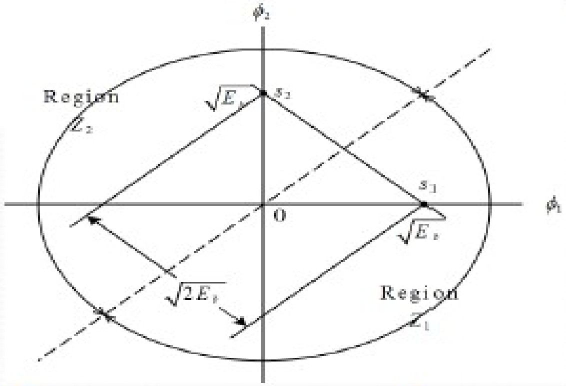
$$= \frac{1}{\sqrt{2\pi N_0}} \int_0^\infty \exp\left[-\frac{(y+\sqrt{E_b})^2}{2N_0}\right] dy$$

- Let $\frac{y+\sqrt{E_b}}{\sqrt{2N_0}} = z$ then $p_{10} = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/2N_0}}^\infty \exp\left(-\frac{z^2}{2}\right) dz$

- Similarly, we may show the p_{01}, the conditional probability of error given that symbol 1 was sent.
- Accordingly, averaging p_{10} and p_{01} and assuming equiprobable symbols, we find that the average probability of bit error or, equivalently, the BER for binary FSK using coherent detection is

$$P_e = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

BFSK



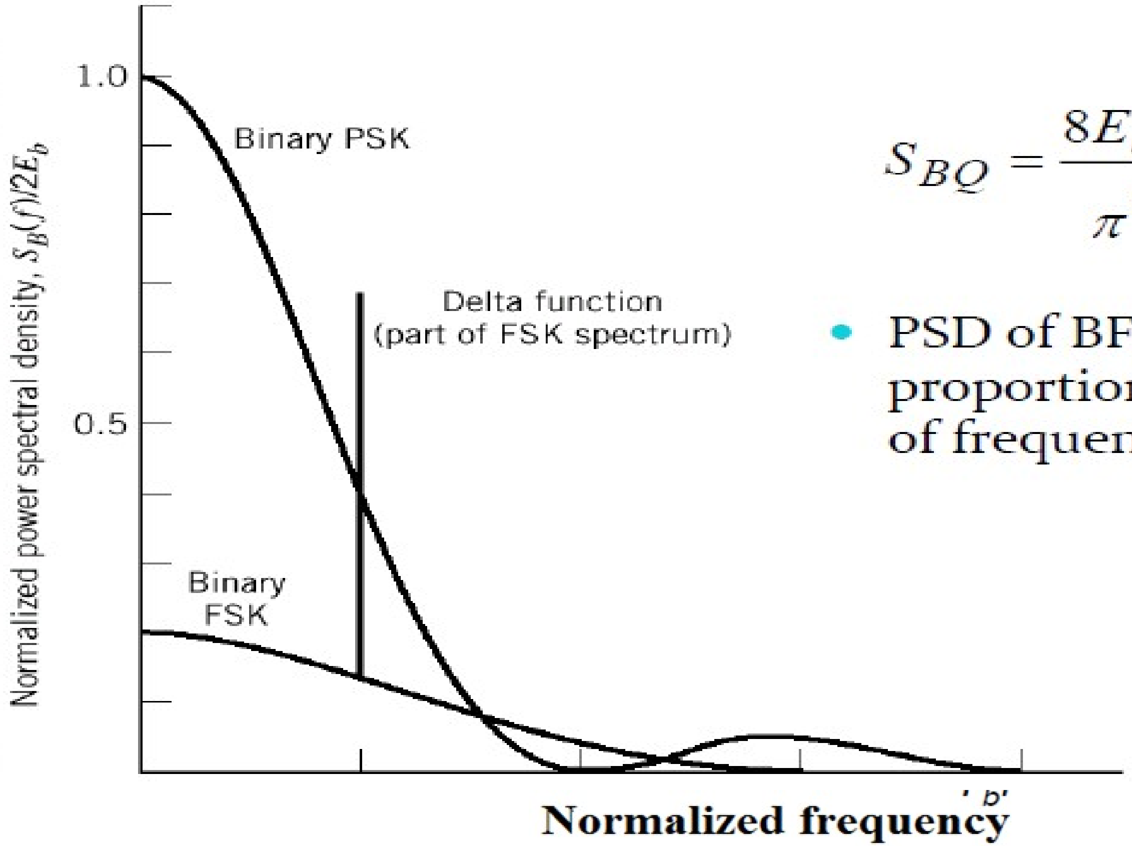
- Euclidean distance is $d_{12} = \sqrt{2E_b}$
- In case of $P_r(0)=P_r(1)$, the probability of error is given by

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\frac{d_{ik}}{2\sqrt{N_0}}\right) \qquad P_e = \frac{1}{2}erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

- *Probability of error P_e of BFSK system is more than the BPSK system.*
- We observe that at a given value of P_e, the BFSK system requires twice as much power as that BPSK system.

Power Spectral density of BFSK

- Finally, we obtain $S_B(f) = S_{BI}(f) + S_{BQ}(f)$



$$S_{BQ} = \frac{8E_bT_b\cos^2(\pi T_b f)}{\pi^2\left(4T_b^2 f^2 - 1\right)^2}$$

- PSD of BFSK is inversely proportional to fourth power of frequency

Differential PSK (DPSK)

- DPSK can be viewed as the non-coherent version of PSK
- Phase difference between two successive bit intervals is independent of theta
- Phase synchronization is eliminated using differential encoding
- Differential PSK
  - Instead of finding the phase of the signal on the interval $0<t\leq T_b$. This receiver determines the phase difference between adjacent time intervals.
  - If "1" is sent, the phase will remain the same
  - If "0" is sent, the phase will change 180 degree.

The relationship between the binary sequence and its deferentially encoded version is shown in table for a assumed data sequence 0 0 1 0 0 1 0 0 1 1 1.

Binary Data		0	0	1	0	0	1	0	0	1	1
Differentially Encoded Data	1	0	1	1	0	1	1	0	1	1	1
Phase of DPSK	0	π	0	0	π	0	0	π	0	0	0
Shifted Differentially encoded Data d_{k-1}		1	0	1	1	0	1	1	0	1	1
Phase of shifted Data		0	π	0	0	π	0	0	π	0	0
Phase Comparision Output		-	-	+	-	-	+	-	-	+	+
Detected Binary Seq.		0	0	1	0	0	1	0	0	1	1

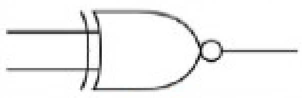
- Or we have binary "1"

$$s_1(t) = \begin{cases} \sqrt{\dfrac{E_b}{2T_b}}\cos(2\pi f_c t); & 0 < t \le T_b \\ \sqrt{\dfrac{E_b}{2T_b}}\cos(2\pi f_c t); & T_b < t \le 2T_b \end{cases}$$

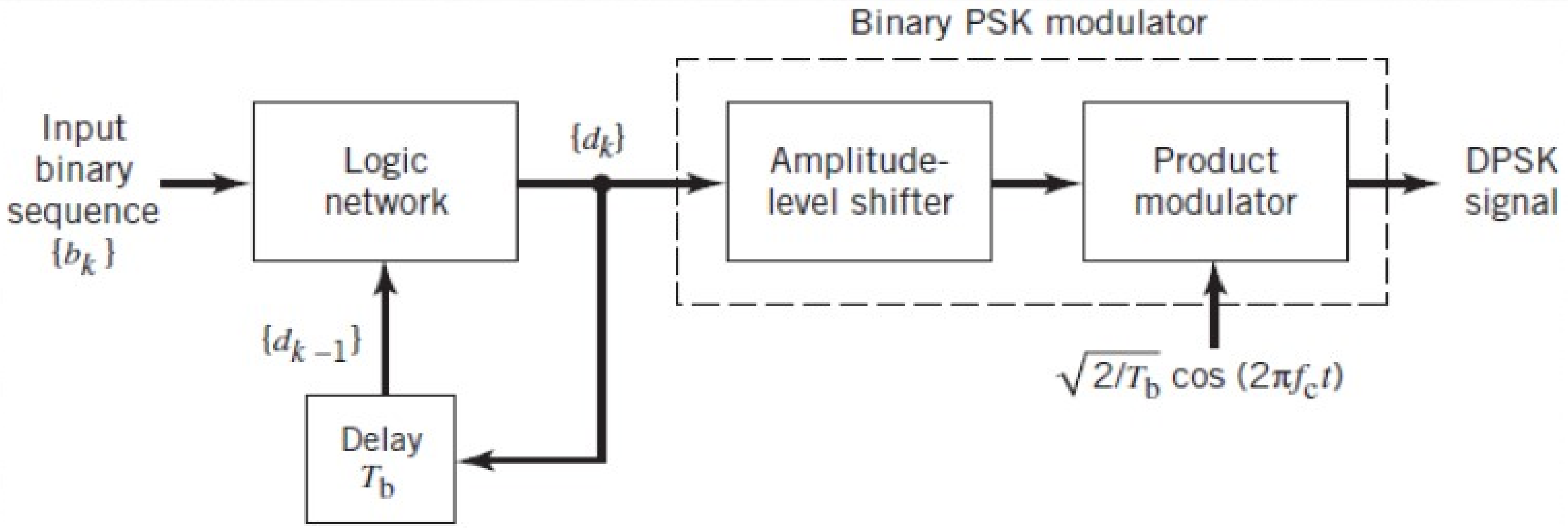
- And binary "0"

$$s_2(t) = \begin{cases} \sqrt{\dfrac{E_b}{2T_b}}\cos(2\pi f_c t); & 0 < t \le T_b \\ \sqrt{\dfrac{E_b}{2T_b}}\cos(2\pi f_c t + \pi); & T_b < t \le 2T_b \end{cases}$$

DPSK: Transmitter



$$d_k = b_k d_{k-1} \oplus \bar{b}_k \bar{d}_{k-1}$$

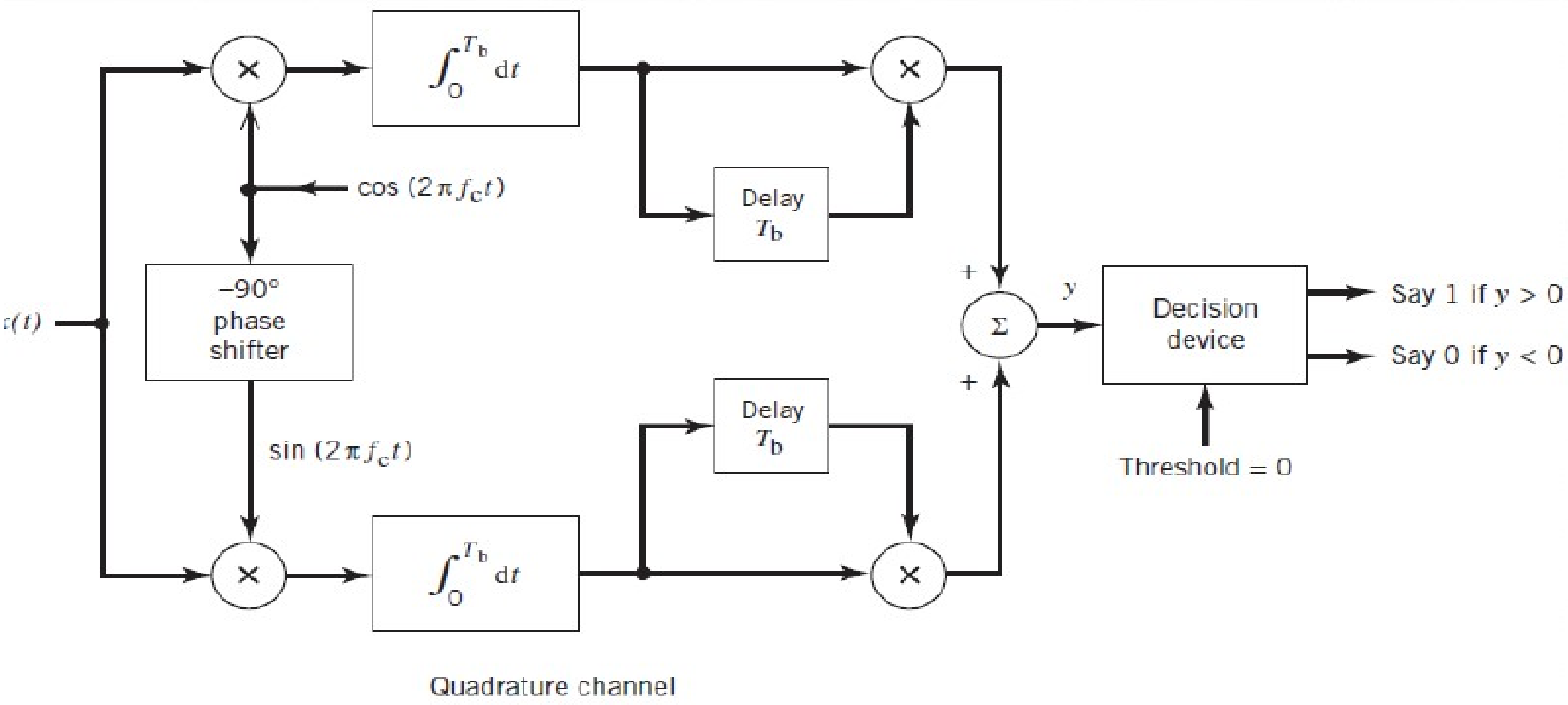


Block diagram of a DPSK transmitter.

- Exclusive-NOR logic

DPSK: Receiver

Phase + + − + + − + − − −



Block diagram of a DPSK receiver.

- In this case, we have $T=2T_b$ and $E=2E_b$
- Hence, the probability of error is given by

$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$$

Detecting regular BPSK needs a coherent detector, requiring a phase reference

DPSK needs no such thing. The only reference is the previous bit which is readily available

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